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Using GeoGebra with Van Hiele's Model in Geometry Classroom: An Experience with Prospective Teacher

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ABSTRACT

Various advances in technology, communication, and information have provided great benefits to all users around the world, especially in digital communication and learning mathematics in the classroom. However, without a systematic and clear model, the use of technology becomes vague and ineffective in cultivating students' interest and understanding in the field of Science, Technology, Engineering, and Mathematics (STEM). This study aimed to explore prospective teachers' (PT) understanding and experience using GeoGebra with van Hiele's model in the geometry classroom. The research employed a qualitative approach, specifically a case study design, which included semi-structured one-on-one interviews and classroom observations. Additionally, dynamic GeoGebra task completion, digital objects, tests, and electronic surveys were also administered. The bottom-up data analysis process was conducted inductively by reading the collected data and forming segments, determining the codes, and filtering overlapping codes until themes were identified. The results showed PTs' sequential understanding using GeoGebra occurred enthusiastically in a supportive environment. This understanding was evidenced when students engaged in activities such as examining, analysing, doing, comparing, identifying, explaining, recognising, distinguishing, and solving tasks. Furthermore, the experience of PTs using GeoGebra involved a collaborative environment, motivation to learn, awareness of various technologies, and independent learning. In conclusion, the transformation of PT's comprehension using GeoGebra in Mathematics is actively demanded with the use of a guidance process such as van Hiele's model. It provides greater chances for students' engagement while delivering more appropriate and meaningful learning.

1. Introduction

The integration of technology in mathematics education is often associated with STEM [1]. This Australian researcher raises concerns that there is still much to be done to highlight the practice of technology integration in education in higher, secondary and primary education institutions. Among their suggestions is to use a project-based teaching and learning approach to enable meaningful integration to occur. In Larkin and Lowrie [1] systematic review among Primary Students, Malaysia is

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ranked sixth in the location of STEM studies. This means that the government is very committed to ensuring integration technology into education in Malaysia is comparable to other developed countries [2].

Technology – integrating into education: The concept of integrating technology into education can be defined as a combination of various media in a digital environment to fulfill not only student learning style but also the pedagogy and curriculum design purposes [3]. Therefore, 21st learning environments should consider preparing future generations with technology skills in teacher training institutions [4]. However, appropriate use of theory [5] and the procedure of using tool creatively and effectively [6] are the challenges in integrating technology into mathematics educations.

Mathematics education: Mathematics education is the practice of teaching and learning mathematics [7] through problem solving that involves learning algorithms and formulas needed for calculations. It is a platform to learn and teach mathematics in a better way [8]. One of the content do-mains in mathematics education including an assessment in Trends in International Mathematics and Science Study (TIMSS) 2019 is geometry [9]. Therefore, geometry is an important concept. Besides, it is all around us and related to the science of measuring Earth and its properties [10].

Geometry: Geometric concepts are commonly represented as total entities rather than as components or attributes [11,12]. However, previous researchers pointed out that learning geometry not only develops spatial sense, but also improves mathematical thinking [13]. Therefore, Mohamed and Kandeel [14] developed a program to teach children to compare between the names and properties of geometric shapes in their study. Additionally, to improve pre-service and in-service teachers' mathematical thinking, Hall and Chamblee [15] applied GeoGebra in their study.

GeoGebra: GeoGebra does not not only enhance Geometry thinking [16], but it also facilitates teaching and learning between teachers and students. In fact, it is a powerful application in the visualization of abstract content, and in facilitating 2D and 3D digital object explorations [17].

Previous researches involve that applied GeoGebra in prospective teachers study include Tatar and Zengin [18], Hähkiöniemi *et al.*, [19], and Haciomeroglu *et al.*, [20]. However, Tatar and Zengin [18] and Haciomeroglu *et al.*, [20] studies focused on prospective secondary mathematics teachers.

Hence, this study examines the use of GeoGebra in geometry classroom by prospective elementary mathematics teachers who majoring in national language. Our focus is to explore their practice in the use of GeoGebra in geometry classroom. Based on van Hiele model of five level of geometric maturity, prospective teachers' understanding and experience were explain contextually. Among various theories or models in geometry education, van Hiele model was chosen as it emphasises the sequential manner [21]. It helps students to construct geometry concept from basic to highest level. The successive level namely visualization, analysis, informal deduction, deduction and rigor are represented as van Hiele model [11].

The methodology of the research is a qualitative approach; whereby a case study was employed. Meanwhile, subject of the study is a group of college students aged 20 years old who are prospective elementary mathematics teachers in a teacher ed-ucational institute in one of East Asia countries. Two weeks class meeting of four learning sessions using van Hiele sequences level was held physically. There were aligned with the students' course description requirements. Data collection was executed using electronic survey, classroom observation, face to face interview and test. Thus, triangulation of the data obtained from various instruments was possible.

In the next sections, historical, development, and criticisms aspects about the van Hiele model, integrated with GeoGebra is further discussed. It is followed by methodology section explaining the context of study to overcome barriers in utilising van Hiele model. Finally, the results are elaborated and suggestions for the future practice are presented.

2. Van Hiele Theory

The van Hiele model has attracted previous researchers such as Crowley [11] and Sharma [21] and stated its origin from the doctoral works of Dina van Hiele-Geldof and Pierre van Hiele at University of Utrecht. Focusing on mathematical concepts among 12 years old students, the couple developed sequential theory for explaining the development of the geometrical concepts [21]. The worked was continued by Pierre after Dina passed away, later gaining international attention when geometry curriculum was revised in the 1960s [21]. By 1984, some of the couple's major works were translated into English by Fuys *et al.*, [22].

Some major criticisms aspects about the van Hiele model were discussed by Sharma [21]. Those include the sequential development, difficulty in identifying students' level, misconception occurrence at different stages, not focusing on non-Euclidean geometry, and constrained approach to the role of language in geometric concept improvement. However, the theory has been accepted until today since it aligns with the Piagetian theory of cognitive development and constructivist approach as the students' improvement depends on each level. Thus, their progress is influenced by prior experiences, knowledge and comprehension at the previous level [21].

In current study, the teaching strategies using van Hiele's model supported by Vygotsky sociocultural paradigm. It is applied since the activities are developed based on students' active participation. Vygotsky [23] emphasizes on the relationship between the subject and object of knowledge which influences the cultural context. Meanwhile, proximal zone development concept is considered as the distance between the real level development, determined by solving problem competence, and the potential development level under adult guidance or in collaboration with more capable others. Therefore, the role of social interaction with peers is crucial for the psychological, cognitive and affective developments that allows better understanding level of geometric thinking. In addition to social relations, mediation through instrument, such as the GeoGebra tool, allows students' development in topics associated with geometry through collaborative learning. Collaborative learning, according to Houghton *et al.*, [24] is to expand learning and knowledge. In addition, Piaget's cognitive development theory was referred as GeoGebra allowing students to adapt to the challenging factors such as contradictions, difficulties, and imbalances in learning [25]. Therefore, this study employed GeoGebra with Van Hiele's model in geometry classroom of the prospective teachers as explained in section 3. There are five sequential levels in the van Hiele theory which include:

Level 0: Visualization

Level 1: Analysis

Level2: Informal deduction

Level 3: Deduction

Level 4: Rigor

2.1 Five Levels in Van Hiele's Model

2.1.1 Level 0: Visualisation

Geometric objects are recognised as a total entity and shape [11,26] without considering their properties. At this level, students are able to classify or differentiate based on the appearance of figures [27,21].

1.1.2 Level 1: Analysis

The students analyse figures in relation to components and connection between the components and recognise properties/rules from a group of shapes empirically [21]. For example, geometric symbols such as A and A' are used when verbalize the solution [27]. However, the definitions have not yet been mastered [26].

1.1.3 Level 2: Abstraction/ Informal deduction

The students logically relate properties/rules in the previous stage by giving informal arguments [21]. For example, they discover properties of symmetry or symmetrical figures and prove them by assembling informal deductive arguments by means of known definitions and properties. Not only, they are able to use different definitions for the same concept, but they can also discover and understand the relationship between products [27].

1.1.4 Level 3: Deduction

The students reason formally complete with undefined terms, axioms, an underlying logical system, definitions, and theorems [28]. The algebraic structure can be understood, properties of the group can be discovered, and students can prove them by means of formal proofs [27].

1.1.4 Level 4: Rigor

Theorems in different postulation systems are established and analyzed [29]. At this level, the students understand the way mathematical systems are established and can use various evidence. They comprehend Euclidean and non-Euclidean geometry. They have described the effect of adding or removing an axiom on a given geometric system [30].

1.2 Five Levels in Van Hiele's Model

GeoGebra is originated from Markus Hohenwarter's master thesis at the University of Salzburg in 2002. It is a dynamic mathematics software suitable for all education levels. It brings together different digital tools such as geometry, spread-sheets, graphing, statistics, and algebra in one engine. Narh-Kert and Sabtiwu [31] conducted action research involving 150 mathematics education students and in-service teachers from two public universities in Ghana. The students were given pre and post test about their understanding with regard geometric concept. The findings show that employing GeoGebra method in lessons is more practical and easier for students to understand. Besides, there is an improvement in the interest of the learning and teaching Geometry including student score. Thus, they recommended that GeoGebra should be incorporated in teaching and learning Geometry [31]. Two years before, there were 23 secondary school mathematics teachers participated in the mixed method study at a small US university. To evaluate teachers' abilities in using GeoGebra, lesson construction were checked, van Hiele test score were used, and written explanation were analysed. The results show that teachers' misconceptions became obvious while visualizing conjectures in the software [32].

Both studies utilised GeoGebra and Van Hiele's model for both students and teachers, which had created meaningful learning. However, they do not explain in detail how GeoGebra is used by prospective teachers' in the context of higher education institutions in depth. A study conducted by

Jaafar *et al.*, [33] mentions the GeoGebra application which is one of the most effective applications in improving student understanding in universities and helping educators interact with students, but does not explain how this application can be used to create students' interaction and improve understanding. Therefore, the research questions are as follow:

- i. What are the prospective teachers' understanding about GeoGebra application in geometry classroom?
- ii. How the prospective teachers experience using GeoGebra in geometry classroom?

2. Methodology

2.1 Data Collection and Analysis

This study applied qualitative approach which is a case study to explore prospective teachers' understanding and experience in employing GeoGebra in geometry classroom. The data were collected using instruments such as interviews, digital objects, document, and survey. First, semi-structured interviews ranging between 15 to 45 minutes were conducted one-to-one basis by the lead author from early February to late March 2023. To ensure the interview questions were receptive, able to produce rich responses and targeting both research questions [34], it was piloted to several prospective teachers. The data was also collected from digital objects consist of submitted tasks by the students via the padlet or google classroom platform. Apart from that, the document was obtained from student test answers. Finally, the survey was employed via digital platform such as Google Form. The respondents, both students and the prospective teachers need to select their answer from five-point Likert scale with the following options: 1. for strongly agree; 2. for agree; 3. for undecided; 4. for disagree; 5. for strongly disagree. The questionnaire comprises of 30 questions to answer the research questions and meet the objectives of the study.

Data analysis process was conducted in inductive way from bottom-up; whereby the researchers read all the collected data and divided into segments. Segments of text that shared common theme were labelled (coding), the overlapping code was combined together until a set of themes were formed [35]. The codes/ themes were generated based on related theories, literature review and researchers' experience as educators. Since Malay language is widely spoken and understood, the interview data was collected in the national language. However, the interview transcripts were analysed before a translator translated the emerged concepts and categories to English language. Afterwards, back translate was applied and the final translation had to be checked and approved by an expert committee as recommended by Chen and Boore [36].

A group of 17 college students in teachers training institute in one of the East Asia countries participated in the study. They were all 20 years old and were pursuing a major in language study, with a minor in mathematics for their first degree programme. In addition to exploring research questions, this study aimed to enhance students' interest in mathematics and assess their recall of geometry knowledge acquired during secondary school. To ensure research ethics, the participants' identities were concealed using nicknames (pseudonyms) and anonymisation. Therefore, the prospective teachers were referred as PT1, PT 2, and so on. In this article, the terms "prospective teacher", "PT" or "students" were used interchangeable to refer to the study participants.

In the course descriptions, lecturers are required to integrate any Geometry applications in their classes. Consequently, GeoGebra application was selected due to its constructions availability for students, its ability to optimize learning time [17], and its suitability for understanding and experiencing geometric properties, as well as helping to develop the Geometry concept. In the Geometry class, students utilized the GeoGebra software for four sessions, amounting to three credit hours. The GeoGebra activity spanned over two weeks, with classes being held twice a week.

Subsequently, a test was administered, and during the students' free time, an interview session were conducted.

2.2 Application of Van Hiele's Model in Geometry Classroom

In the geometry classroom, the students engaged in activities in groups of three. They were assigned two tasks to visualize transformations in the visualization stage. First task required students to choose a type of transformations in a plane, such as translation, reflection, glide reflection, or rotation, and create a drawing using GeoGebra. In the second task, students observed a figure before and after the transformation and had to explain the procedure using GeoGebra. The development of understanding, following van Hiele's model, occurred in sequential manner. This approach provided prospective teachers with an opportunity to recall their prior geometry knowledge from school, while students with higher level of understanding were able to guide their peers with a lower level of understanding.

Therefore, in the analysis level, students analysed the use of software and performed simple tasks and basic steps. Some mistakes appeared in their graphic sketches. In the informal deduction/ abstraction level, students created sketches in GeoGebra based on their understanding, verbalized the transformation accordingly, compared their graphic pictures with those of other groups, identified their mistakes, corrected their sketches for the next task, and explained the procedures systematically while providing reasons to their drawings. During the second and third levels, students' misconceptions were overcome, and the use of mathematical language was developed through explanations and exchanging ideas in group discussions.

In the deduction level, students recognized the process of finding complete figure and the type of transformation they and other groups have done, specifically for the second task. Furthermore, students were able to distinguish their group's sketch from those of other groups. Therefore, in the informal deduction and deduction stages, students were required to explain and use language. Regarding the criticism of not focusing on non-Euclidean geometry, this study aimed to utilise the GeoGebra software, allowing students the opportunity to explore non-Euclidean and Euclidean geometry as they experimented with different icons in GeoGebra applications.

Table 1
Van Hiele's Model in the context of study

Level	The context of study
1 Visualization	Students examine two given tasks to visualise the transformation
2 Analysis	Students analyse the use of software Students perform simple tasks/ basic steps Some mistakes appear in their graphic sketch
3 Informal deduction/ abstraction	Students create some sketches in GeoGebra based on their understanding Students verbalise the transformation accordingly Student compare their graphic pictures with those of other groups Students identify their mistakes and correct the sketches for the next task Students explain the procedure systematically and provide reasons for the drawings
4 Deduction	Students recognize the process of finding complete figure and the type of transformation they have done and those of other groups Students are able to distinguish their group sketches from those of other groups
5 Rigor	Students understand and solve the test properly

In the final stage namely rigor, students were presented with problems in the form of a test and were required to utilise their understanding and experience from all levels to solve the problems. Table 1 summarises the procedure of the study's context using van Hiele's model.

3. Results

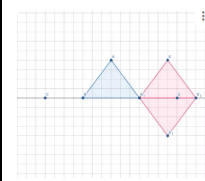
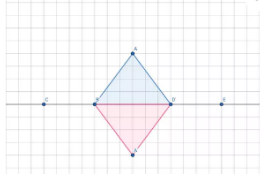
3.1 The Understanding of Prospective Teachers in the use of Geogebra in Geometry Classroom

The understanding of prospective teachers in the use of GeoGebra was assessed through various means, including classroom observations, digital submissions of tasks via platforms like Padlet or Google Classroom, interview sessions, and documents such as tests and students' tasks (refer to Table 2). Thirteen categories (refer to Figure 1) were identified, demonstrating prospective teachers' understanding and aligning with the five levels in Van Hiele's Model. The first category is students choose any type of transformation they prefer and visualize it using GeoGebra. The second category is students observe the initial and complete figures in the second task and identify the types of transformation involved the process of transforming a 2D picture into 2D digital representations. The third category is students characterize the location and the function of icons in GeoGebra.

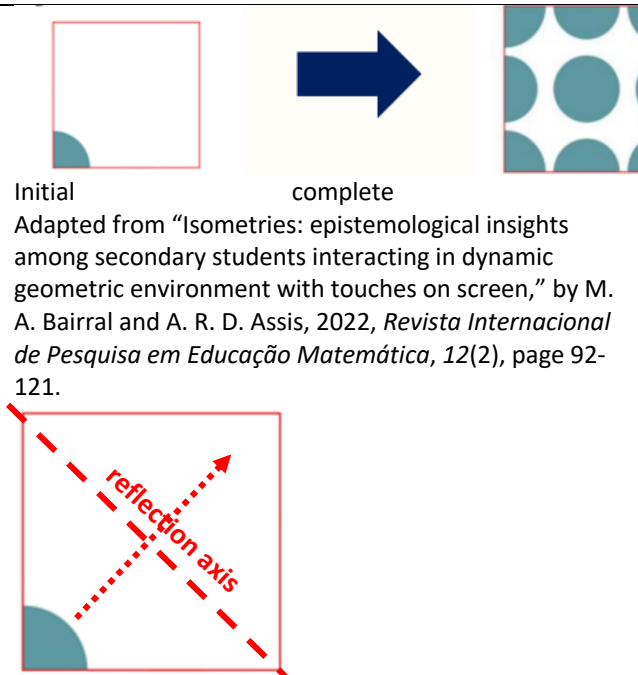
The fourth is students differentiate between icons that are easy and difficult to understand. Whilst the fifth category is students create sketches in GeoGebra based on their understanding; the sixth category is students verbalize the transformation accordingly. Followed by seventh and eighth category which are students identify the mistakes in transformation geometry and students explore the internet to learn more about the function of each icon respectively. Meanwhile, the ninth category is students refine their knowledge by seeking help from peers and lecturers when encountering difficult-to-understand icons or misconception. The next two categories are students present an overall step- by-step process and a complete figure and students accurately compare their sketches with those of other groups. Finally, the last two categories are students solve problems systematically on paper based on their prior knowledge and students recognise the cause-and-effect relationship between the GeoGebra sketch and the sketch on paper. The explanation van Hiele's model in the context of study is presented in Table 2 and summarised in Figure 1.

Table 2

Explanation Van Hiele's Model in the context of study

Level	The context of study	Prospective teachers' understanding while using GeoGebra	The data from interview transcripts, documents and digital objects
1 Visualization	Students examined two different tasks to visualize the transformation	First, students selected a type of transformation they preferred and visualized using GeoGebra Then, students observed initial and complete figures in the second task and identified the types of transformation	Students did several transformation such as: 1) glide reflection  2) Reflection 

occurred in the transformation process from 2D pictures into 2D digital representations



Initial complete
Adapted from “Isometries: epistemological insights among secondary students interacting in dynamic geometric environment with touches on screen,” by M. A. Bairral and A. R. D. Assis, 2022, *Revista Internacional de Pesquisa em Educação Matemática*, 12(2), page 92-121.

2 Analysis

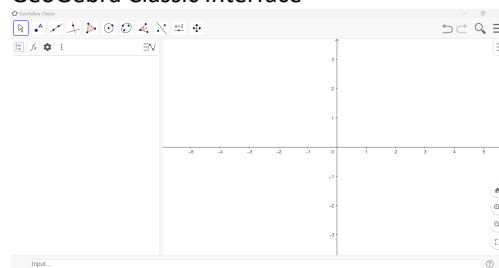
Students analysed the software utilization

Students performed simple tasks
Some mistakes occurred in their graphic sketches

Students characterised the location and the function of icons in GeoGebra

Students differentiated the icons which were easy and difficult to understand

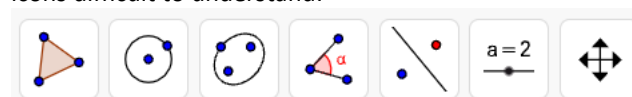
Sample of icon located at the upper left side in GeoGebra Classic interface



icons easy to understand:

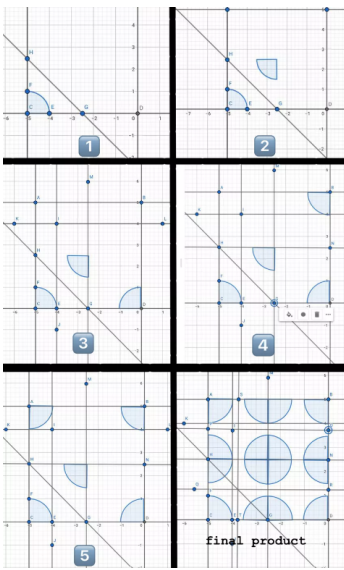
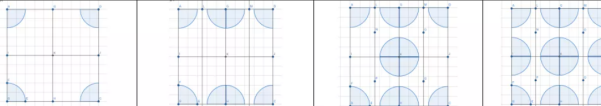
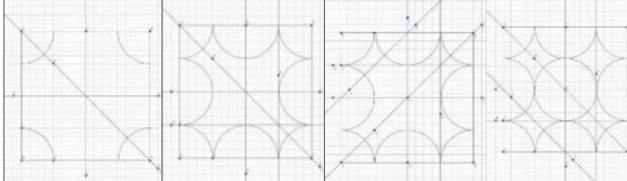
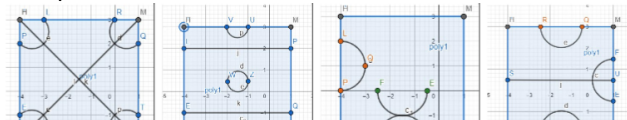


icons difficult to understand:



“I tried the icons that looked simple, I retried the easy icons that had a clear meaning and would be able to see what actually the icon was. I tried the hard ones and also chose to identify the particular icons myself then I did the exploration. ...” (PT7)

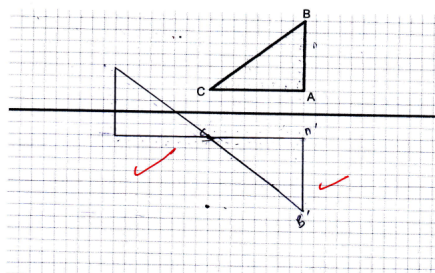
Students used an alphabet to represent each vertex, for example ABC and A'B'C'

4 Deduction	<p>Students recognized the process of finding complete figure and the type of transformation they and other groups have done</p> <p>Students could distinguish their group's sketch from those of other groups</p>	<p>Students demonstrated an overall step-by-step process and presented a complete figure.</p> <p>Students compared their sketches accurately with those of other groups</p>	<p>Group PT1</p> 
			<p>Group PT5</p> <p>"Process for completing figure for PT1 group and PT5 group quite similar. ..." (PT1)</p>
			<p>Group PT4</p> 
			<p>Group PT3</p>  <p>"Process for completing figure for PT4 group and PT3 group are the same. ..." (PT4)</p>
			<p>Group PT2</p>  <p>"Process for completing figure for PT2 group quite different from PT4 and PT3 group. ..." (PT2)</p>
5 Rigor	<p>Students understood and solved</p>	<p>Students solved the problems on paper</p>	PT5

the test
properly.

systematically
based on prior
knowledge.

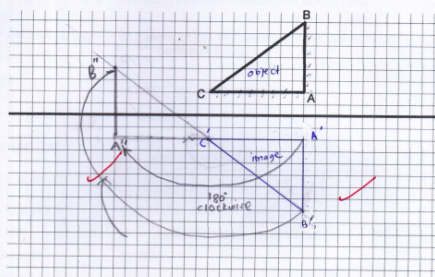
6. Show the final image after the following set of transformation:
Reflected in the 'mirror' line and then rotated 180° clockwise about the image of C.
Label the image appropriately and explain the process.



Explanation:
The first you find the mirror transformation
object A at in the mirror line and over
a object to create a triangle B is reflected.
After that and the point want to use to
rotated the triangle clockwise to in that
diagram the point is both rotation on point
C.

PT1

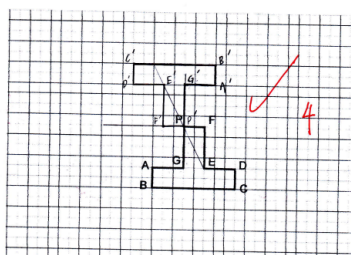
6. Show the final image after the following set of transformation:
Reflected in the 'mirror' line and then rotated 180° clockwise about the image of C.
Label the image appropriately and explain the process.



Explanation:
- Firstly, the object must be reflected on the mirror line. This is called
reflection. The image must be congruent.
- All the image we labeled it with the symbol (') prime.
- For example A becomes A', B becomes B' and C becomes C'.
- The image must be the same size.
- After that, from the image of C which is C', we marked it as
center of rotation.
- Connect all the image point A', B' to the center of rotation.
- Make a rotation symbol which is clockwise.
- Rotate the point using 180° clockwise.
- Relabeled the new point with (')
- For example A' to A'', B' to B'', C' to C''
- The image must be the final image will be congruent because
it involved reflection transformation and the rotation which is called composite

PT4

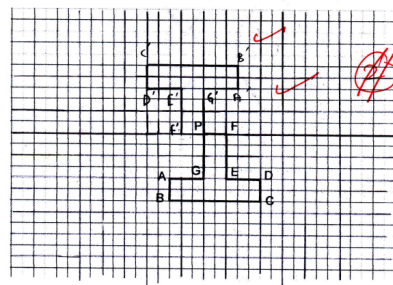
5. Show the image of the following shape after a rotation about point P by 180° (half
turn) in a clockwise direction. Label the image appropriately and state whether the
image is congruent to the object. Explain the process.



Explanation:
The image is congruent.
Firstly, I draw a straight line from point P on point P to form an
image of point P'. I do a straight line because it is stated that
the angle must be 180° which is half of a full rotation. Since point P
is the nearest to the center of rotation (P) and the closest to find the
image, so I made image start the rotation with point P.
Next, I draw a straight line from point G as I draw it on for
point P before. After making a line, I count the total of the small
square from point G to point P to know the exact location of image G.
Then, the process is repeated for point E. After determining these 3
images which are P', G' and E', I draw a complete image of the
object to form a new shape that has been rotated 180° in a clockwise
direction at point P.

PT3

5. Show the image of the following shape after a rotation about point P by 180° (half turn) in a clockwise direction. Label the image appropriately and state whether the image is congruent to the object. Explain the process.



Explanation:
Image 2 is a congruent with image 1. It has the same size. It rotated by 180° and will be half on the line because 180° is half ~~from~~ 360° .

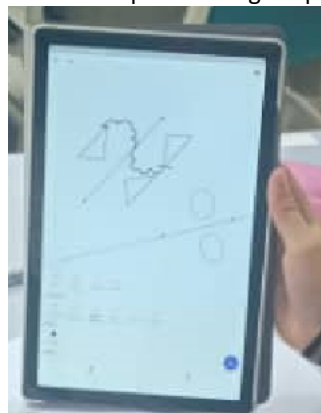
"Need to be more detailed.. from the aspect of angles and shapes to utilise GeoGebra.. Detailed information needs to be acknowledged to get more accurate shapes and precise outcomes. ..." (PT3)

"The GeoGebra is more detailed and more accurate than normal drawings. It doesn't take much time to use..in just a few minutes you can finish drawing geometrical shapes. ..." (PT2)

Students recognized the cause and effect relationship between GeoGebra sketch and their sketches on paper.

"If we make a mistake when using the GeoGebra, we can instantly redo or delete, unnecessarily to use the eraser. It also has a grid.. It's up to us to use it. When we make use of the grid, it looks clearer and neatly arranged. ..." (PT1)

"GeoGebra does not provide instructions to the meaning of each icon, that means when we point the icon, its didn't pop up the meaning of icons. Secondly, GeoGebra does not have pages...so we have to reorganize the arrangement of the content in particular. One more thing.. GeoGebra requires a device with a relatively large screen.. so it is kind of burdensome to use a device with a small screen because it is quite tough to move the icon, therefore I prefer using a laptop. ..." (PT7)



A student showed a different sketch on the same page in GeoGebra interface

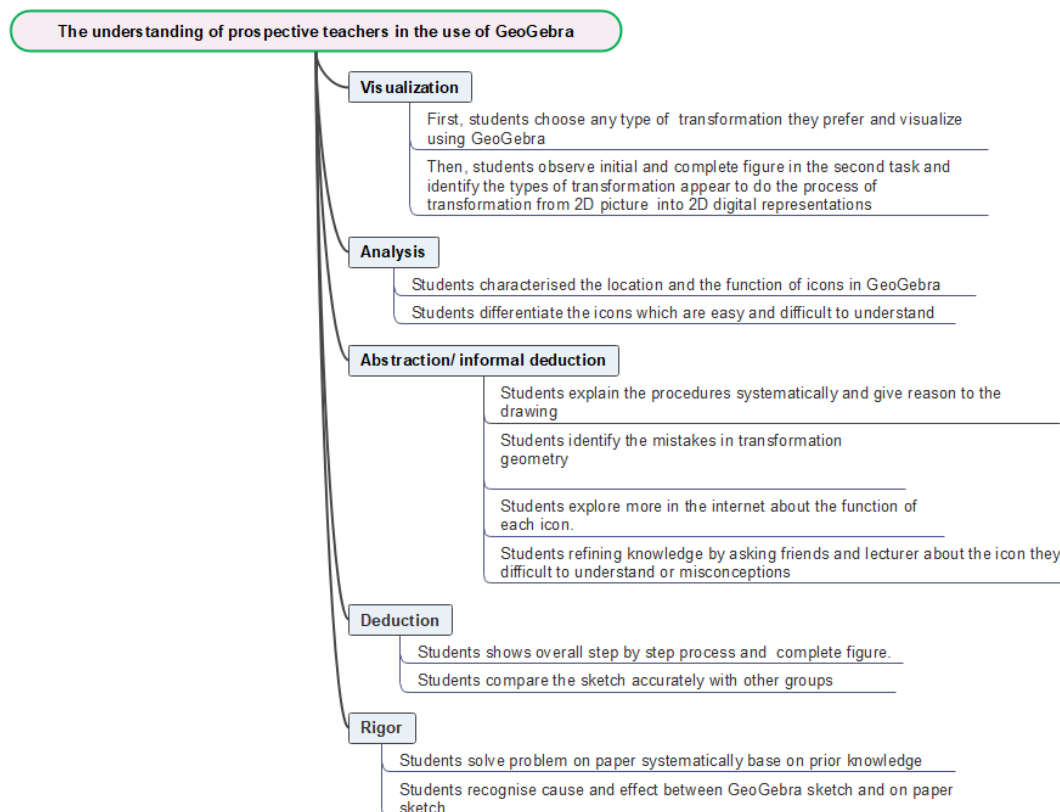


Fig. 1. Summary of prospective teachers' understanding in the use of GeoGebra

3.2 The Experience of Prospective Teachers using Geogebra in Geometry Classroom

Based on the observation in class, interviews, and digital objects such as electronic survey conducted, there are four categories of experiences reported by the prospective teachers regarding the use of GeoGebra in Geometry classroom. Those include collaborative learning environment, motivation to learn, creating students awareness of various technologies in mathematics learning, and encouraging independent learning.

3.1.1 Collaborative learning environment

Prospective teachers discovered that using GeoGebra encouraged them to collaborate in groups when working on tasks (Figure 2).

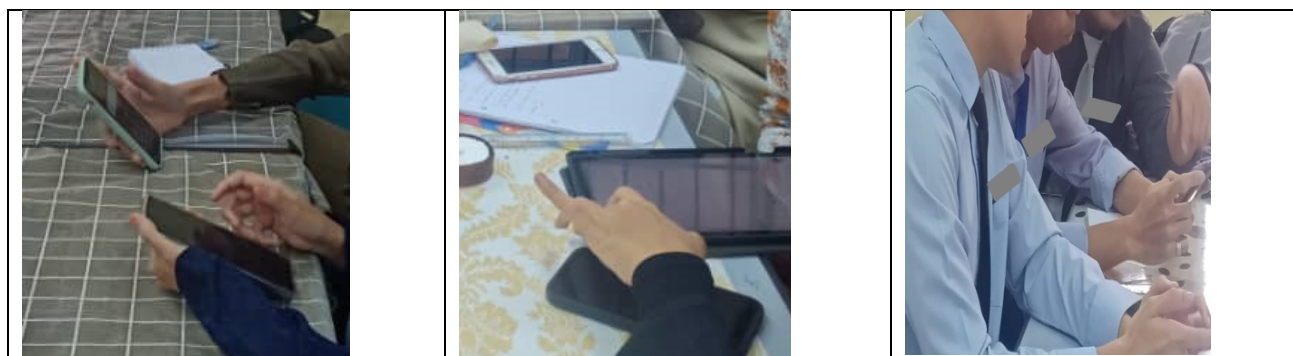


Fig. 2. Prospective teachers work in a group when using GeoGebra

From the electronic survey, prospective teachers' were asked to indicate their level of agreement regarding whether GeoGebra encouraged them to discuss with friends. The results showed that 52.94% of the students agreed, 41.18% strongly agreed and 5.88% were undecided. These findings indicate that learning mathematics using GeoGebra provides students with an opportunity to communicate with friends and promotes active student participation in class.

These results were further supported by the interview session, where prospective teachers mentioned their tendency to seek help from friends and lecturers when they encountered difficulties during the task. One prospective teacher stated:

If there was an icon that I was not familiar, normally I asked my friends to help. Anyhow instead of asking, I chose to try to do it on my own. (PT1)

3.1.2 Motivation to learn

Regarding the questionnaire on students' enjoyment and satisfaction, the students' responses indicate that 52.94% agreed they enjoyed using GeoGebra in the Geometry class, 35.29% choose strongly agreed and 11.76% were undecided. Additionally, after using GeoGebra apps, 64.71% of students agreed that they realised mathematics was fun, 29.41% strongly agreed and only 5.88% disagreed. In terms of students' perception of GeoGebra usability the findings showed that more than half prospective teachers (58.82%) agreed it was easy to use, 23.53% strongly agreed and 17.65% are undecided. According to Ngoepe [37], when individuals engage in activities that they find interesting, satisfying or enjoyable, they experience intrinsic motivation.

Based on the interview data, prospective teachers also described that using GeoGebra enhances their interest in mathematics:

The first positive aspect when we used GeoGebra was that it could attract students' attention during the teaching and learning session, they would enjoy it very much.

Surprisingly the use of GeoGebra was much extensive and broader. We could change and used quadratic equations in it and I felt a bit engrossed to explore more. (PT7)

At first I noticed that GeoGebra was interesting. It made work easier..I found out it was simple to use for a tutorial or self-study. (PT5)

Using GeoGebra for mathematics subject is easier for students in school since there are topics that require writing and drawing angles. (PT2)

When there is something that is unchallenging, we feel happy to do it. Even I used at the last minute, it did not feel like it was difficult because it seemed quite simple and took me a short time to complete the task. It was fun and pleasing experience, I never knew this existed before, but when I downloaded it, I felt like there was a productive tool. It was an excitement because there were learning aids available to help me learn. (PT4)

Besides, prospective teachers mentioned that using GeoGebra increases their motivation to learn. One prospective teacher stated:

GeoGebra motivates me to learn and do tutorials. (PT4)

Another mentioned:

GeoGebra motivates me to investigate and explore more things related to mathematics. (PT7).

The results showed that using technology in mathematics not only involved cognitive elements but also emphasised the motivational aspect. The process and learning experience have a positive effect on students, increasing their intrinsic motivation. This aligns with the findings of Picard *et al.*, [38], which highlight the importance of the learning process and its effect on students' motivation.

3.1.3 Create students' awareness of various technologies in mathematics learning

The interview data revealed that some prospective teachers became aware of using various technology tools while using GeoGebra. They mention that:

It is ubiquitous and movable since we can open it using phone or tablet or laptop. (PT4)

I usually use my laptop and phone to use the GeoGebra application. (PT6)

If you want to see the result you can print .. but before that, save using pdf in your laptop. (PT5)

From the submitted tasks on the Padlet platform, prospective teachers utilized the drawing tool to create various shapes (Figure 3). They also had the option to select different backgrounds for the GeoGebra interface, including grid lines, small-scale grid lines, or larger scale grid line (Figure 4). Additionally, the GeoGebra apps provided a colouring tool with different colours, which attracted prospective teachers to use it creatively (Figure 5).

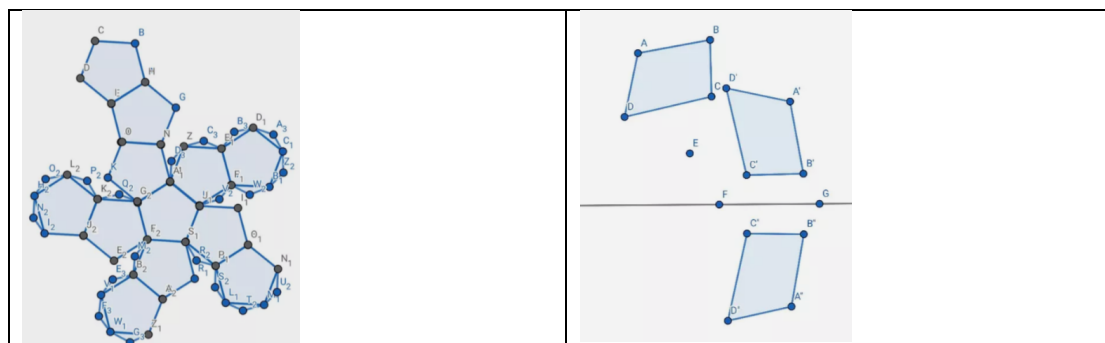


Fig. 3. Drawing different shape using GeoGebra

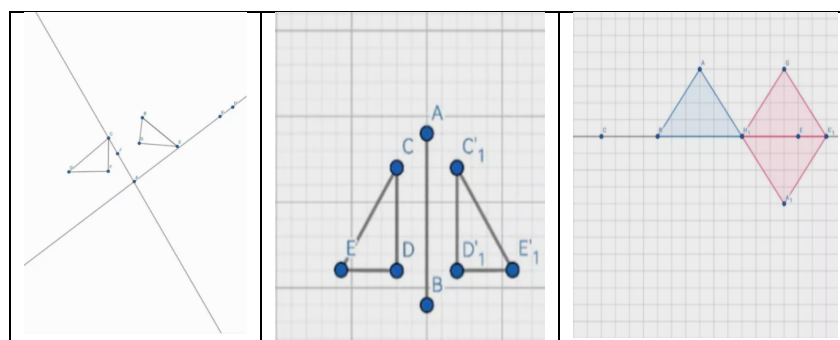


Fig. 4. Different backgrounds with and without grid lines

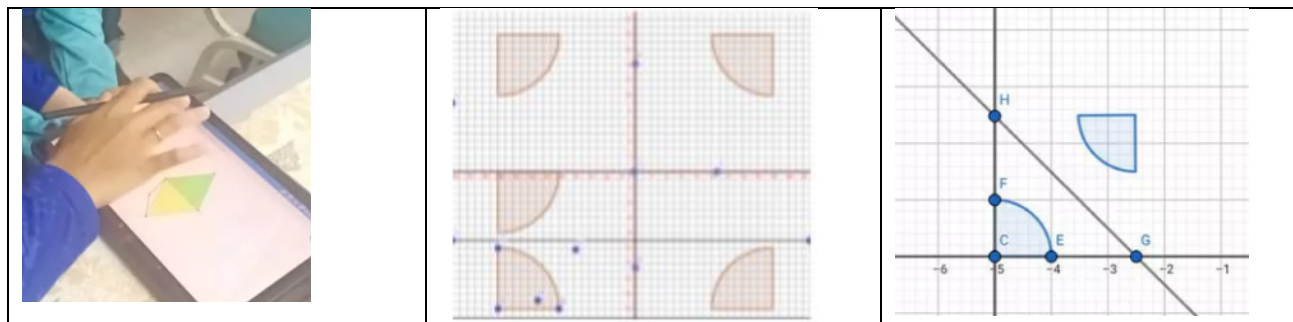


Fig. 5. Different colour provided

Regarding the various technology tools provided, prospective teachers realised that were acquiring new technology skills when using GeoGebra:

My technology skills are growing. (PT5)

To me GeoGebra is a new application for learning... so I am more interested in using GeoGebra because it has the position of the icons inclusive of images and pictures in it.. I am more interested in exploring every function in GeoGebra.. so indirectly it will improve my skills to understand each icon in GeoGebra. (PT6)

GeoGebra increases our desire to use technology...It sharpens our talent and skills to explore every icon and function in GeoGebra, so we are not out of date. (PT6)

Using GeoGebra not only suite with students' multiple learning styles such as verbal and visual representations, but also integrated with various technology tools and skills that provide students opportunity to learn as well as mathematics topics in a more dynamic manner [17].

3.1.3 Encourage independent learning

During the interview session, some prospective teachers acknowledged that they used GeoGebra apps for self-paced learning anytime and anywhere:

It can be used everywhere even in a moving car we can use our phone. (PT2)

While another stated:

It is ubiquitous and portable since we can open it using phone or tablet or lap-top. (PT4).

The questionnaire regarding the use of GeoGebra via smartphones showed that 35.29% of prospective teachers agreed with using GeoGebra on smartphones, 29.41% strongly agreed, 23.53% disagreed and 11.76% strongly disagreed. When asking students about their preference for using laptops and tablets instead of smartphones, they mentioned that they preferred tools with a wider screen to easily performed tasks in GeoGebra.

With the ability to mobilize GeoGebra via smartphones, tablets or laptops, most most prospective teachers were encouraged to explore on their own before seeking help from friends or lecturers. In terms of promoting independent learning, some participants mentioned the following:

At first I will explore each and every icon. (PT6).

My urge inside me is to explore more and more...GeoGebra. (PT3).

I am more interested to probe more. (PT7).

I tried first the icons that was easy for me to comprehend, then I tried again since the easy icons had a clear meaning and we could see what it was. For the hard ones I tried and retried to identify more and then explored by myself. (PT7).

At first I tried to click the icon myself before indulging in my own experiment. I would rather do it on my own before asking somebody. After some successive effort I would do multiple exploration again and again. (PT1)

GeoGebra doesn't have pages...so we have to organize it ourselves...in terms of the arrangement of the content, if we want to make any geometrical shape, we have to rearrange it ourselves. (PT7)

As claimed by de Sousa *et al.*, [17], GeoGebra tasks promote students' active engagement in their own learning and enhance learning conditions.

4. Conclusions

Considering Van Hiele's model in geometry classrooms provides prospective teachers with a clear understanding and meaningful experience in the learning process of using the GeoGebra application. The experience of prospective teachers starts from the basic level of visualization and analysis and progresses to more complex stages such as abstraction, deduction, and rigor. This progression aligns with Bloom's [39] definition of understanding, which involves translation, interpretation, and extrapolation. Therefore, this study can provide knowledge about the effective use of technology, especially GeoGebra in mathematics classes even if it takes place face-to-face or virtual. Teachers also benefit when there are learning guidelines in conducting project-based learning by integrating technology. With that, there is no reason that teachers are not confident in planning learning as stated in the study of Jamaluddin *et al.*, [40].

Besides, using GeoGebra in mathematics classrooms implies that Van Hiele's model serves as an important guideline for students' tasks. However, a detailed explanation of each level in the procedure should consider the learning context, students' abilities, and the grade level. Additionally, prospective teachers' experience in using GeoGebra has encouraged them to explore the technology further and has instilled enthusiasm for its application not only in other subjects but also in future teaching endeavours. Moreover, the dynamic use of GeoGebra via smartphones, tablets or laptops stimulates interest in technology skills and mathematics, encourage using English as the language in explaining the process, cutting across different fields specifically among prospective teachers majoring in national language. Future studies need to see what and how mathematical language elements are applied through the use of GeoGebra among prospective teachers to increase communications that impact children's mathematical practices when prospective teachers begin teaching in school, as mentioned in Yusoff *et al.*, [41].

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