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SIR-SI Dengue Model: Evaluating Control Strategies for Disease Mitigation

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ABSTRACT

Dengue is one of the infectious diseases and has become the most public health issue. The previous studies were focused on the mathematical modelling of dengue transmission dynamics with different factors such as climates, mobility, time delays, and others. However, many existing models do not comprehensively assess the combined effects of multiple control strategies on dengue transmission, leading to gaps in effective intervention planning. By gaining a more detailed understanding of the relationship between dengue transmission dynamics and its control strategies, the spread of dengue disease could be diminished. In this research, the method proposed is mathematical modelling and estimation of reproduction number on the dengue transmission with the implementation of control strategies. The dengue model consists of SIR-SI compartments which are SIR for human populations and SI for mosquito populations respectively. The mathematical analysis is carried out including equilibrium, stability analysis and positivity solutions. The basic reproduction number is defined to study the effect of the control strategies on the dengue transmission dynamics. The extended dengue is then developed with the implementation of three control strategies such as insect repellents, insecticides and mosquito fish. The result showed that the basic reproduction number is affected after implementing the different combinations of control strategies in the model. The best intervention is to combine all the control strategies to diminish the spread of dengue.

1. Introduction

Dengue is an illness that causes a high fever, severe headache, rashes and muscle and joint pain [1]. Dengue virus is one of the common mosquito-borne viruses from the family of Flaviviridae.

There was a total of four serotypes of dengue virus were known around the world such as DENV-1, DENV-2, DENV-3 and DENV-4. Dengue is widely spread throughout the tropics, with local variations in risk influenced by rainfall, temperature, relative humidity and unplanned rapid urbanization. Dengue virus cannot be transmitted directly from human to human, but the spread of dengue virus

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requires an infected female mosquito bite as a vector. The mosquito species that transmit dengue virus are Aedes Aegypti and Aedes Albopictus. This transmission occurs when a mosquito bites an infected human and later bites a healthy individual, facilitating virus spread [13].

The signs and symptoms of an individual who is infected by dengue virus are depending on the case severity level and it can even be asymptomatic. The course of dengue infection takes off after the incubation period and is divided into three main phases: febrile phase, critical phase (which may include haemorrhagic manifestations or dengue shock syndrome) and recovery phase [2]. Moreover, dengue is also causing high mortality. Thus, to prevent the continuous spreading of dengue virus, there were different approaches suggested and done. For example, dengue vaccination, mosquito control by using pesticides, biological control by implementing the food chain and prey-predator concept in the ecosystem and others. Infectious disease modelling plays a crucial role in public health strategy and epidemic forecasting [12,14]. Mathematical models incorporating vector-host interactions have also been applied in dengue transmission studies, providing useful insights for understanding and controlling disease dynamics [16].

Different studies have been done by researchers in recent years. They have focused on how to develop the dengue model, investigated the factors that influence the transmission of dengue disease and with different optimal controls [3-8]. The authors lacked focus in comparing different combinations of control strategies and investigating the impact of the control strategies on the behaviour of the basic reproduction number. The authors lacked of define the relationship between model parameters and the basic reproduction number. The dengue model parameters should be studied based on the basic reproduction behaviour, so that the implementation of different control strategy is functional.

While numerous studies have explored dengue transmission modelling and optimal control strategies, a clear gap remains in evaluating the combined effectiveness of multiple control measures within a unified framework. Existing models often isolate specific factors such as climate, mobility or single interventions without integrating them holistically. Therefore, this study aims to fill this gap by examining the impact of combining insect repellents, insecticides and mosquito fish in reducing dengue transmission, particularly through their effects on the basic reproduction number.

Section 1 is the introduction or background of dengue. In section 2, the model structure of dengue transmission dynamics is described including the generation of the basic reproduction number. Section 3 presents the extended dengue model with three optimal controls, along with the analysis of optimal control is described. Section 4 contains the numerical simulation that illustrates the dynamics results. Finally, section 5 presents the conclusion.

2. Methodology

2.1 Basic Dengue Model

There were two populations which are human and mosquito populations and it is denoted as N_H and N_V respectively. For human population, it is subdivided into three compartments, susceptible human, infected human and recovered human while for mosquito population, it is subdivided into two compartments, susceptible mosquito and infected mosquito.

Based on the idea and studies presented by Andraud *et al.,* [9] and Hethcote [10], the basic dengue model is developed based on the assumptions as follows. Figure 1 shows the flow diagram of *SIR-SI* dengue model while Table 1 shows the descriptions of the variables and the descriptions of the parameters for the basic dengue model respectively. The model parameters were chosen based on relevant literature and assumptions reflective of general dengue transmission characteristics. However, further elaboration on the estimation and sensitivity of these parameters, particularly for control variables u_1, u_2 and u_3 would enhance model transparency and reproducibility. Future iterations could also consider a systematic sensitivity analysis to better understand the influence of each parameter on the reproduction number.

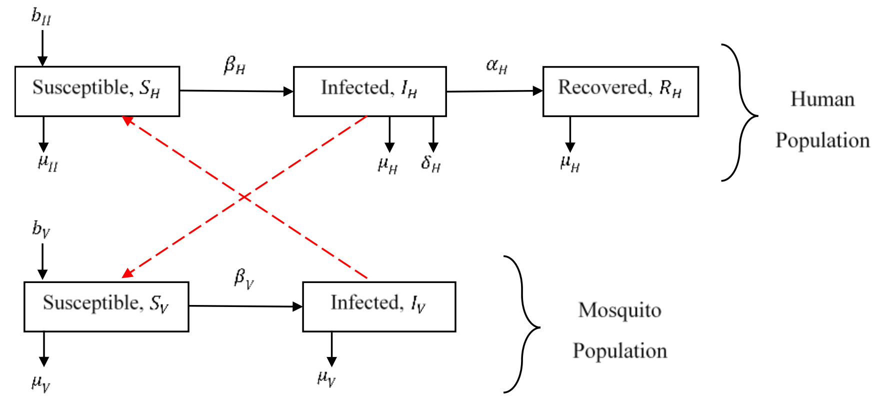

Fig. 1. Flow diagram of basic dengue model

Table 1
Variables and parameters of basic dengue model

Variables/Parameters	Descriptions
S_H	Number of susceptible humans
I_H	Number of infected humans
R_H	Number of recovered humans
S_V	Number of susceptible mosquitoes
I_V	Number of infected mosquitoes
N_H	The total human population
N_V	The total mosquito population
b_H	Recruitment rate for humans (birth and immigration)
μ_H	Natural death rate for humans
δ_H	Disease induced death rate for humans
β_H	Human contact rate
α_H	Recovery rate for humans
b_V	Recruitment rate for mosquitoes (birth and immigration)
μ_V	Natural death rate for mosquitoes
β_V	Mosquito contact rate

2.1.1 Human population

The spread of dengue in total human populations at time, t are divided into susceptible human $S_H(t)$, infected human $I_H(t)$ and recovered human $R_H(t)$. Thus Eq. (1):

$$S_H(t) + I_H(t) + R_H(t) = N_H(t) \tag{1}$$

The number of susceptible humans are increasing via the recruitment rate of human by birth or immigration into the population, b_H, while decreasing through the natural death rate, μ_H and the infective rate, $\beta_H I_V S_H$ after human contact with the infected mosquito. Hence Eq. (2):

$$\frac{dS_H}{dt} = b_H - \mu_H S_H - \beta_H I_V S_H \tag{2}$$

The number of human infected increasing after contact with infected mosquito at infective rate, $\beta_H I_V S_H$ undergoes an incubation period of 4 to 10 days. The number of human infected decreasing due to the recovery rate, α_H disease-induced death rate, δ_H and natural death rate, μ_H. Thus Eq. (3):

$$\frac{dI_H}{dt} = \beta_H I_V S_H - \alpha_H I_H - \mu_H I_H - \delta_H I_H \tag{3}$$

The number of recovered people increased as the number of infected people getting recover at recovery rate, α_H while decreased at the rate of natural death, μ_H. Hence Eq. (4):

$$\frac{dR_H}{dt} = \alpha_H I_H - \mu_H R_H \tag{4}$$

2.1.2 Vector population

The spread of dengue in total human populations at time, t are divided into susceptible mosquito $S_V(t)$ and infected mosquito $I_V(t)$. The vector component of the model does not include an immune class as mosquitoes never recover from the infection, that is their infected period ends with their death due to their relatively short life cycle. Thus Eq. (5):

$$S_V(t) + I_V(t) = N_V(t) \tag{5}$$

The number of susceptible mosquitoes are increasing via the recruitment rate of mosquito by birth or immigration, b_V into the total mosquito population, while decreasing through the natural death rate, μ_V and the infective rate, $\beta_V I_H S_V$ after mosquito contact with the infected human. Hence Eq. (6):

$$\frac{dS_V}{dt} = b_V - \mu_V S_V - \beta_V I_H S_V \tag{6}$$

The number of mosquitoes infected increasing after contact with infected human at infective rate, $\beta_V I_H S_V$ undergoes an incubation period of 24 hours. The number of mosquito infected decreasing due to the its short life cycle or natural death rate, μ_V. Thus Eq. (7):

$$\frac{dI_V}{dt} = \beta_V I_H S_V - \mu_V I_V \tag{7}$$

In short, by considering the above assumptions, the notations of model variables and model parameters and formulating together Eq. (1) to Eq. (7) with the block diagram in Figure 1, the resulting system of a non-linear differential system describing the dengue model in the human and mosquito population are as follows in Eq. (8):

$$\begin{aligned}
\frac{dS_H}{dt} &= b_H - \mu_H S_H - \beta_H I_V S_H \\
\frac{dI_H}{dt} &= \beta_H I_V S_H - \alpha_H I_H - \mu_H I_H - \delta_H I_H \\
\frac{dR_H}{dt} &= \alpha_H I_H - \mu_H R_H \\
\frac{dS_V}{dt} &= b_V - \mu_V S_V - \beta_V I_H S_V \\
\frac{dI_V}{dt} &= \beta_V I_H S_V - \mu_V I_V
\end{aligned} \tag{8}$$

From the model obtained in Eq. (8), it clearly shown the interaction between the susceptible, infected, recovered in human population in (8_1-8_3) with change in time. Similarly, the interaction between susceptible and infected mosquito populations in (8_4-8_5) with change in time.

2.2 Basic Reproduction Number

The basic reproduction number is defined as the estimated number of secondary infections originating from a single person during the infectious period. From the description by Diekmann *et al.,* [11], the expression of basic reproduction number can be found by using the next generation operator approach. Theoretically, the basic reproduction number is the critical value that determines the dengue disease spread or die out. For example, if $R_0 > 1$, the disease will continue to spread in a specific area and R_0 is the scale plate of the spread rate; the disease will extinct when $R_0 < 1$. By using the next generation operator approach, the basic reproduction number is defined as in Eq. (9).

$$R_0 = \pm\sqrt{\frac{\beta_H b_H \beta_V b_V}{\mu_H \mu_V^2 \left(\alpha_H + \mu_H + \delta_H\right)}} \tag{9}$$

From this, the relationship between the parameters and the basic reproduction number, it can be quantified that higher values of β_H, β_V, b_V and lower value of μ_V will allow the outbreak of dengue disease. Conversely, for small values of β_H, β_V, b_V and large value of μ_V will bring the disease dies out. The reproduction number is a powerful parameter which measures the existence and stability of the disease in the human and mosquito population. If $\beta_H b_H \beta_V b_V < \mu_H \mu_V^2 \left(\alpha_H + \mu_H + \delta_H\right), R_0 < 1,$ the disease-free equilibrium is the only equilibrium and then the disease dies out. If $\beta_H b_H \beta_V b_V > \mu_H \mu_V^2 \left(\alpha_H + \mu_H + \delta_H\right), R_0 > 1,$ the unique endemic equilibrium exists and the disease persists with the population.

3. Extended Dengue Model

The basic dengue model Eq. (8) is then extended with three optimal controls. For susceptible human population, the rate of infected by contact to infected mosquitoes is denoted as

$(1-u_1)\beta_H I_V S_H$ where $u_1(0 \le u_1 \le 1)$ is the control on the application of insect repellent. Repellent-based interventions have been studied for their measurable impact on dengue transmission rates [18]. Similarly, for susceptible mosquito population, the rate of infected by contact with infected human is denoted as $(1-u_1)\beta_V I_H S_V$. Despite that, u_2 is the use of insecticide spray to increase the death rate of mosquito and $(0 \le u_2 \le 1)$. u_3 is the biological control with mosquito fish which feed mosquito larvae as foods. Thus, the recruitment of mosquito population is controlled via $(1-u_3)b_V$ and $u_3(0 \le u_3 \le 1)$. Figure 2 illustrates the three main control strategies, u_1, u_2 and u_3, used to mitigate mosquito populations and reduce dengue transmission.

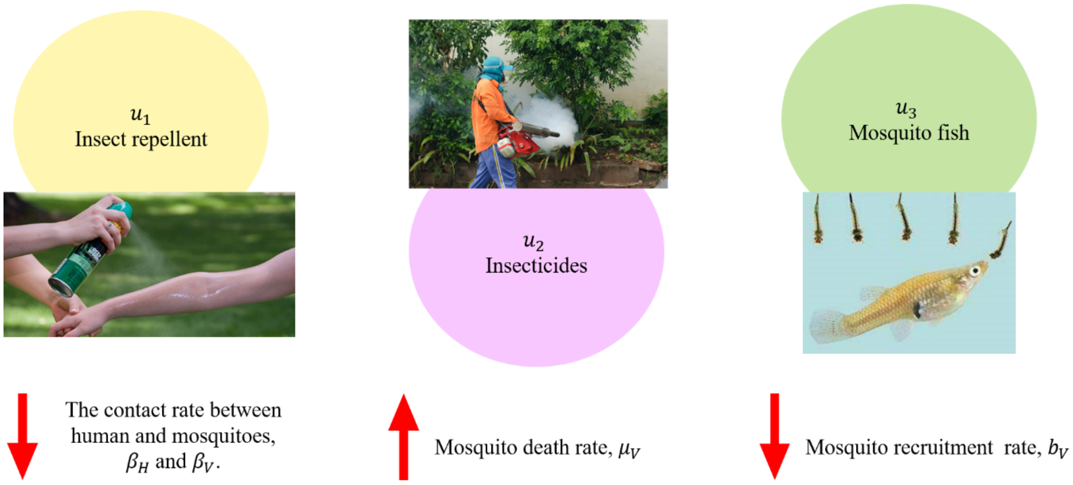

Fig. 2. Controls u_1, u_2 and u_3

Next, by combining all the above assumptions and formulations, the following extended model is presented in Eq. (10):

$$\begin{aligned}
\frac{dS_H}{dt} &= b_H - \mu_H S_H - (1-u_1)\beta_H I_V S_H \\
\frac{dI_H}{dt} &= (1-u_1)\beta_H I_V S_H - \alpha_H I_H - \mu_H I_H - \delta_H I_H \\
\frac{dR_H}{dt} &= \alpha_H I_H - \mu_H R_H \\
\frac{dS_V}{dt} &= (1-u_3)b_V - (\mu_V + u_2)S_V - (1-u_1)\beta_V I_H S_V \\
\frac{dI_V}{dt} &= (1-u_1)\beta_V I_H S_V - (\mu_V + u_2)I_V
\end{aligned} \tag{10}$$

To determine the efforts of the optimal control that probable in desire to control the dengue spread, the optimal control problem with the objective function, J is considered Eq. (11).

$$J(u_1, u_2, u_3) = \int_0^T [a_1 I_H + a_2 I_V + b_1 u_1^2 + b_2 u_2^2 + b_3 u_3^2] dt \tag{11}$$

where T is the final time and the coefficients a_1, a_2, b_1, b_2, b_3 are positive weights to balance the factors. With the given objective function $J(u)$, it is aimed at minimizing the number of infected humans $I_H(t)$ and infected mosquito $I_V(t)$ while minimizing the cost of control strategies $u_1(t), u_2(t), u_3(t)$. Therefore, an optimal control u_1^*, u_2^*, u_3^* is obtained such that in Eq. (12) where the control set U is represent as Eq. (13).

$$J\left(u_1^*, u_2^*, u_3^*\right) = \min\left\{J\left(u_1, u_2, u_3\right) \middle| u_1, u_2, u_3 \in U\right\} \tag{12}$$

$$U = \left\{\left(u_1^*, u_2^*, u_3^*\right) \middle| u_i : [0,T] \to [0,1], \textit{Lebesgue measurable } i = 1,2,3\right\} \tag{13}$$

3.1 The Hamiltonian and Optimality Conditions

In this section, the extend dengue model with optimal control is analysed according to the Pontryagin's Maximum Principe, the Hamiltonian and optimality conditions are defined from the cost functional and the governing dynamics. The adjoint system is introduced by Pontryagin with relation of the differential equation to the objective functional. Thus, the necessary conditions were defined [13].

Note that for Hamiltonian function, the variables $X = \left(S_H(t), I_H(t), R_H(t), S_V(t), I_V(t)\right)$, $U = \left(u_1(t), u_2(t), u_3(t)\right)$, the adjoint variables, $\lambda = \left(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\right)$. The Lagrange function, $L = L\left(I_H, u_1, u_2, u_3, t\right) = a_1 I_H + \sum_{j=1}^{3} b_j \left(u_j(t)\right)^2$. Note that λ_i represents as the adjoint variable functions to be determined suitably by applying Pontryagin's maximum principle, where $i = 1,2,3,4,5$. Thus, The Hamiltonian function H of the optimal control problem is defined as follows Eq. (14):

$$H\left(t, X, U, \lambda\right) = L + \lambda_1 \frac{dS_H}{dt} + \lambda_2 \frac{dI_H}{dt} + \lambda_3 \frac{dR_H}{dt} + \lambda_4 \frac{dS_V}{dt} + \lambda_5 \frac{dI_V}{dt} \tag{14}$$

From Eq. (14), we substitute the Lagrange function, L and each differential equation, $\frac{dS_H}{dt}, {}_2\frac{dI_H}{dt}, \frac{dR_H}{dt}, \frac{dS_V}{dt}$ and $\frac{dI_V}{dt}$ into it. Thus, will get as Eq. (15):

$$H = a_1 I_H + a_2 I_V + \sum_{j=1}^{3} b_j \left(u_j(t)\right)^2 + \lambda_1 \left[\frac{dS_H}{dt}\right] + \lambda_2 \left[\frac{dI_H}{dt}\right] + \lambda_3 \left[\frac{dR_H}{dt}\right] + \lambda_4 \left[\frac{dS_V}{dt}\right] + \lambda_5 \left[\frac{dI_V}{dt}\right]$$

$$\begin{aligned} H = a_1 I_H + a_2 I_V + b_1 u_1^2 + b_2 u_2^2 + b_3 u_3^2 \lambda_1 &\left[b_H - \mu_H S_H - \left(1 - u_1\right)\beta_H I_V S_H\right] \\ &+ \lambda_2 \left[\left(1 - u_1\right)\beta_H I_V S_H - \alpha_H I_H - \mu_H I_H - \delta_H I_H\right] \\ &+ \lambda_3 \left[\alpha_H I_H - \mu_H R_H\right] \\ &+ \lambda_4 \left[\left(1 - u_3\right) b_V - \left(\mu_V + u_2\right) S_V - \left(1 - u_1\right)\beta_V I_H S_V\right] \\ &+ \lambda_5 \left[\left(1 - u_1\right)\beta_V I_H S_V - \left(\mu_V + u_2\right) I_V\right] \end{aligned} \tag{15}$$

3.2 Adjoint System and Optimal Control Analysis

Theorem: For an optimal control set u_1, u_2, u_3 that minimizes $J(u_1, u_2, u_3)$ over U, there are adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ satisfying adjoint condition: $\lambda_i^{'}(t) = -\frac{\partial H}{\partial x_i}$, where $i = 1,2,3,4,5$. The Hamiltonian Eq. (14) is then substitute into the adjoint functions. Given the formula, $\lambda_i^{'}(t) = -\frac{\partial H}{\partial x_i} \Rightarrow \lambda' = -(F_x + \lambda g_x)$, where $i = 1,2,3,4,5$. The adjoint functions are defined as follows in Eq. (16):

$$\begin{aligned}
\lambda_1^{'}(t) &= -\frac{\partial H}{\partial S_H} \Rightarrow \lambda_1 \mu_H + (\lambda_1 - \lambda_2)(1-u_1)\beta_H I_V \\
\lambda_2^{'}(t) &= -\frac{\partial H}{\partial I_H} \Rightarrow -a_1 + (\lambda_2 - \lambda_3)\alpha_H + \lambda_2(\mu_H + \delta_H) + (\lambda_4 - \lambda_5)(1-u_1)\beta_V S_V \\
\lambda_3^{'}(t) &= -\frac{\partial H}{\partial R_H} \Rightarrow \lambda_3 \mu_H \\
\lambda_4^{'}(t) &= -\frac{\partial H}{\partial S_V} \Rightarrow \lambda_4(\mu_V + u_2) + (\lambda_4 - \lambda_5)(1-u_1)\beta_V I_H \\
\lambda_5^{'}(t) &= -\frac{\partial H}{\partial I_V} \Rightarrow -a_2 + (\lambda_1 - \lambda_2)(1-u_1)\beta_H S_H + \lambda_5(\mu_V + u_2)
\end{aligned} \tag{16}$$

By doing partial derivatives of the Hamiltonian in terms of u_1, u_2 and u_3, and u_1^*, u_2^* and u_3^* is obtained in Eq. (17).

$$\begin{aligned}
u_1^*(t) &= \max\left\{0, \min\left(1, \frac{(\lambda_2 - \lambda_1)\beta_H I_V S_H + (\lambda_5 - \lambda_4)\beta_V I_H S_V}{2b_1}\right)\right\} \\
u_2^*(t) &= \max\left\{0, \min\left(1, \frac{\lambda_4 S_V + \lambda_5 I_V}{2b_2}\right)\right\} \\
u_3^*(t) &= \max\left\{0, \min\left(1, \frac{\lambda_4 b_V}{2b_3}\right)\right\}
\end{aligned} \tag{17}$$

4. Numerical Solutions

In this section, the numerical analysis of the extended dengue model is presented. The simulation results for the optimal control strategy, which combines all three control measures - insect repellents, insecticides and mosquito fish $(u_1 \neq 0, u_2 \neq 0, u_3 \neq 0)$ are shown in Figure 3 to Figure 7. Similar modelling efforts using daily dengue case data have been conducted in other settings to validate control effectiveness [15]. These figures illustrate the impact on susceptible humans, infected humans, recovered humans, susceptible mosquitoes and infected mosquitoes respectively. By applying all three control strategies to optimize the objective function J, the results indicate significant differences in the state variables compared to the scenario without control measures.

Figure 3 and Figure 4 depict the susceptible and infected human populations under the three optimal control strategies. The number of infected humans initially peaks at 40 thousand but after implementing optimal control, it declines to 27 thousand demonstrating a substantial reduction. Similarly, as shown in Figure 6, the susceptible mosquito population experiences a steep decline within one month of implementing control strategies, whereas without control, the decline occurs gradually over 2.5 months. For infected mosquitoes (Figure 7), the peak population of 50 thousand decreases significantly to 15 thousand, indicating a strong impact of the combined control measures. Additionally, Figure 5 shows an increase in the recovered human population after implementing optimal control, which is a direct result of reducing the number of susceptible and infected individuals in both human and mosquito populations.
The numerical simulation results confirm that the combination of all three control strategies yields the most effective outcome. The implementation of insect repellents, insecticides and mosquito fish significantly reduces the basic reproduction number $R_0 < 1$.

Based on the formula of the basic reproduction number that derived from basic dengue model Eq. (8), the parameters that affecting the value of reproduction number include the recruitment rate, contact rate and death rate for both human and mosquito populations.

The hypothesis made is suggesting three different control strategies, which to raise the death rate of mosquito population, to deduct the recruitment rate of mosquito population and the contact rate between human and mosquito populations. So that the dengue disease is stabilized and the reproduction number, R_0 is always less than 1.

Furthermore, to ensure the basic reproduction number, $R_0 < 1$, several parameters were investigated. The contact rate between human and mosquito populations, β_H, β_V and the recruitment rate for mosquito population, b_V should be always < 1. The recovery rate for human population, α_H and the death rate for mosquito population, μ_V should be always > 1. In other words, β_H, β_V and $b_V < \alpha_H, \mu_V$. The implementation of the three control strategies u_1, u_2 and u_3 which are to control those parameters that affecting the basic reproduction number. It is worth noting that the current model does not account for dynamic environmental or social factors, such as seasonal climate variations, rainfall patterns or population mobility. These external influences play a critical role in the actual spread of dengue and may affect the accuracy of the model's predictions in real-world applications.

These findings demonstrate that an integrated vector control approach effectively reduces dengue transmission and highlights the importance of combining multiple control strategies to achieve the best results.

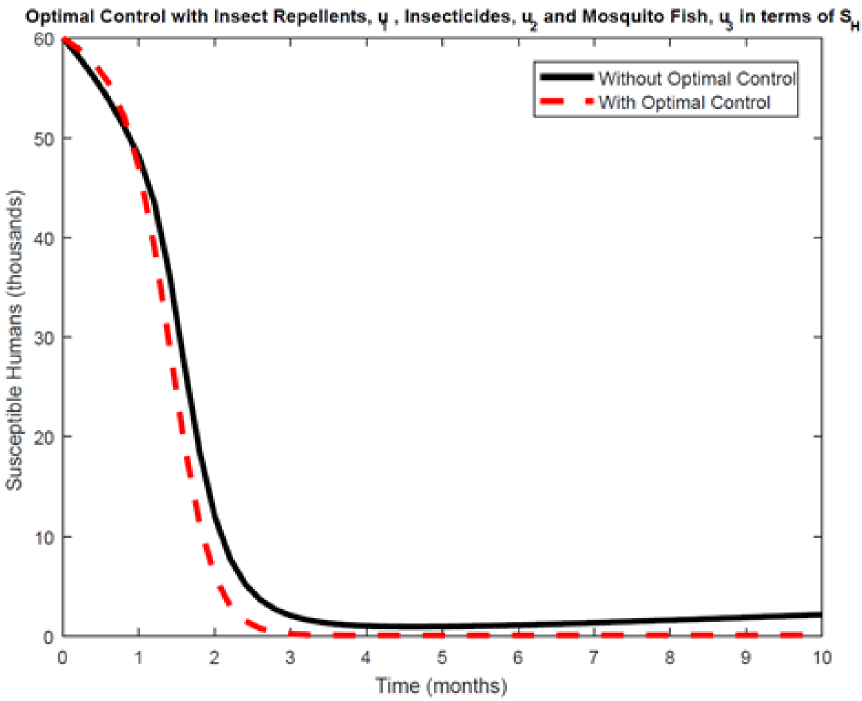

Fig. 3. Susceptible human

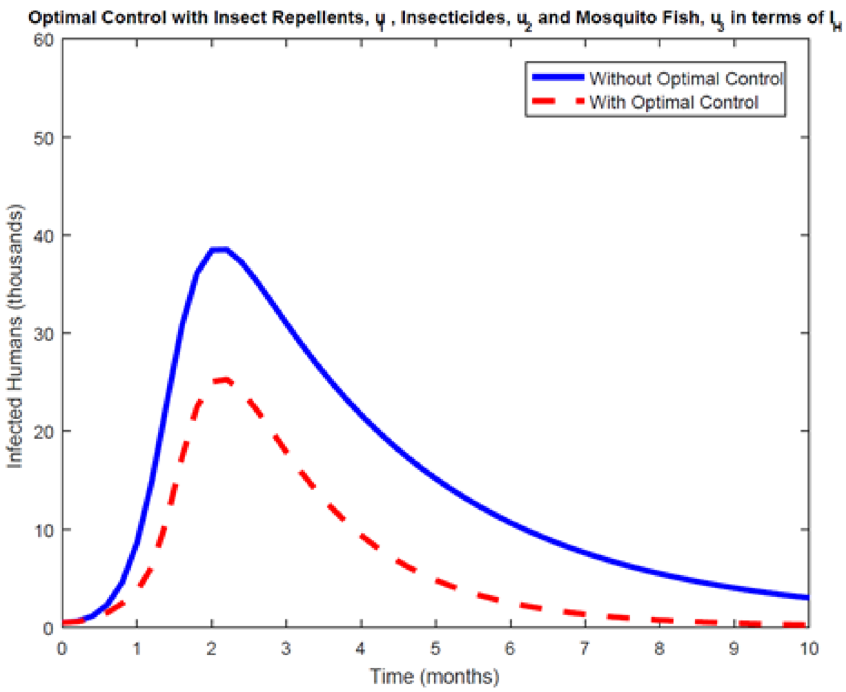

Fig. 4. Infected human

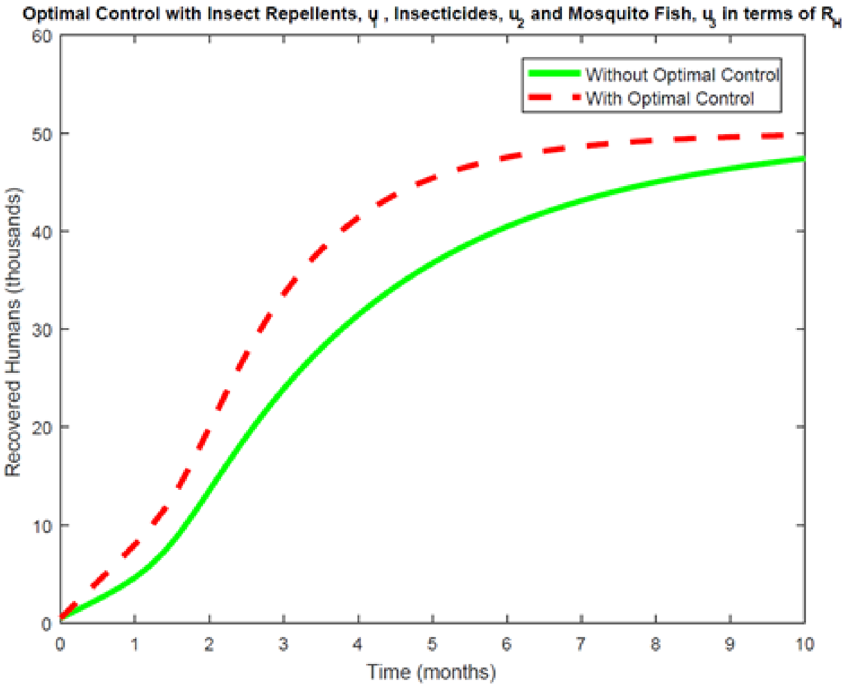

Fig. 5. Recovered human

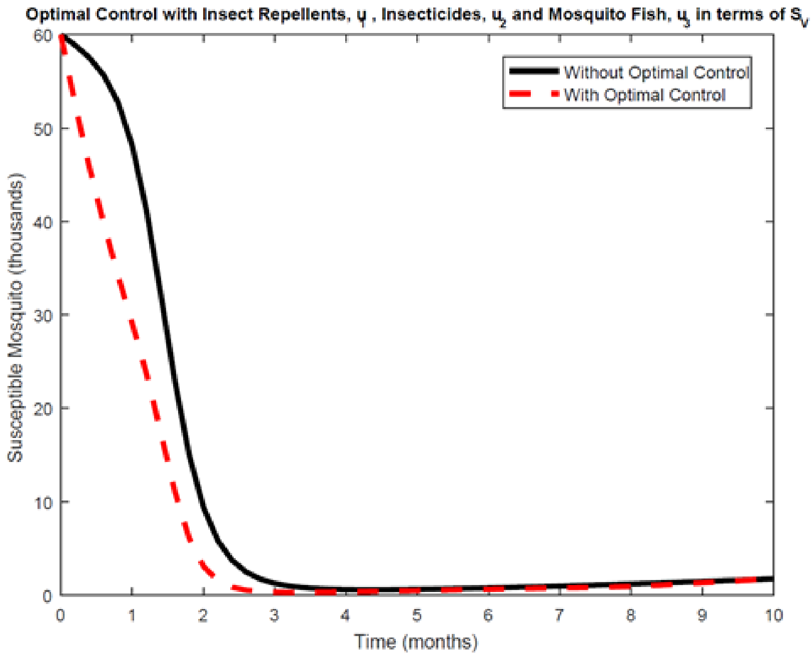

Fig. 6. Susceptible mosquito

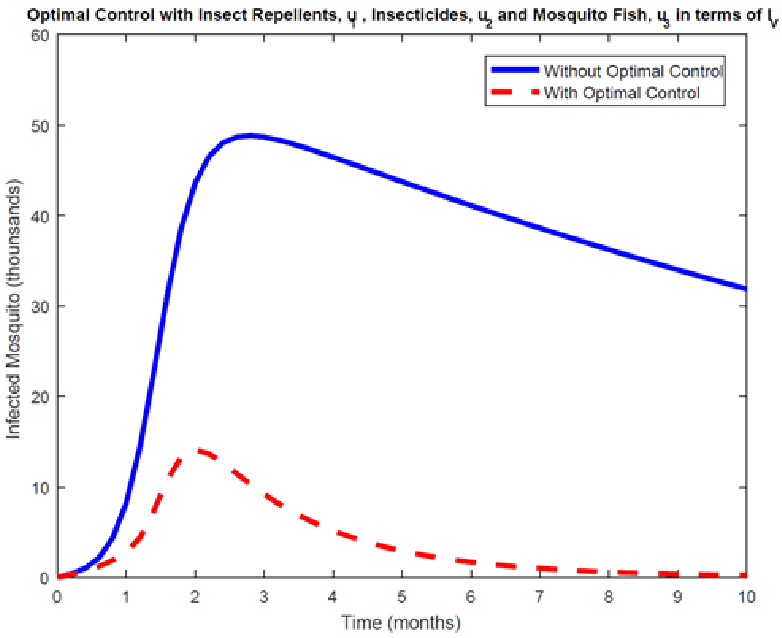

Fig. 7. Infected mosquito

5. Conclusion

In this paper, the basic dengue model, which accurately describes the transmission dynamics of dengue disease was developed. This model was then extended by incorporating three control strategies: insect repellents, insecticides and mosquito fish. The numerical analysis results demonstrated that the combination of all three strategies provides the most effective approach for controlling dengue transmission within a community. Innovative mosquito-repellent formulations with sustained release have shown potential in long-term vector control [17]. The findings indicate that implementing these control measures significantly reduces the number of infected individuals and mosquito populations, thereby lowering the basic reproduction number R_0 below the critical threshold of one.

Furthermore, the study highlights the importance of an integrated vector management approach, as individual control strategies alone may not be sufficient to achieve long-term disease suppression. The results emphasize that a synergistic combination of interventions leads to a more substantial and sustained impact on dengue reduction. By increasing the mosquito mortality rate, decreasing recruitment and minimizing human-mosquito contact, the proposed model provides valuable insights for optimizing public health policies aimed at dengue mitigation.

Despite the promising results, the model's applicability across diverse geographical settings may be limited due to regional differences in mosquito species behaviour, environmental conditions and human-mosquito interaction patterns. Hence, caution is advised when generalizing the findings beyond the modelled scenario without local calibration and validation.

Future research can further enhance this model by integrating real-world epidemiological data and machine learning techniques to predict outbreak trends and optimize intervention strategies dynamically. Overall, this study provides a scientific basis for policymakers and health authorities to design more effective and sustainable dengue control programs, ultimately contributing to public health improvement and disease prevention.

Additionally, integrating real-world epidemiological data would significantly enhance the model's predictive capability and reliability. Incorporating machine learning approaches to analyse outbreak trends and community responses could also provide adaptive, data-driven insights for improving dengue mitigation strategies.

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