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PaSCap : An Interactive MATLAB GUI for Visualizing and Computing a few Parameters of Paraboloid or Spherical-Cap Domes

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ABSTRACT

This paper presents PaSCap, an interactive MATLAB-based graphical user interface (GUI) designed to visualize and calculate key geometric parameters of paraboloid and spherical cap domes. The GUI integrates mathematical modeling with dynamic 3D visualization, allowing users to adjust input variables, observe real-time geometric changes, and obtain accurate values for volume, surface area, and related parameters. PaSCap is intended to support students, educators, and researchers in mathematics, engineering, and architecture by providing an intuitive environment that links equations to visual outputs, thereby enhancing conceptual understanding. Its automated computations and comparative visualizations enable users to explore similarities and differences between the two dome types while gaining deeper insight into their structural and geometric properties. Overall, PaSCap demonstrates how interactive computational tools can simplify complex geometry, enrich teaching and learning experiences, and assist in preliminary analysis and design tasks.

1. Introduction

Curved surfaces such as paraboloids and spherical caps are among the most important three-dimensional (3D) geometries in science, engineering, and architecture due to their favorable structural, optical, and mechanical properties. A paraboloid possesses a unique focal characteristic that allows parallel rays to converge at a single point, making it highly effective in satellite communication systems and solar concentrators, where signal accuracy and energy conversion

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efficiency are critical [1,2]. Beyond engineering applications, paraboloidal surfaces naturally appear in physical phenomena. For instance, the free surface of a rotating liquid assumes a parabolic shape due to the balance between gravitational and centrifugal forces [3]. Accurate computation of geometric properties, such as surface area and volume, is therefore essential in both applied physics and mathematical modeling.

In architectural and structural design, paraboloid geometries are often selected for their visually striking appearance and efficient load distribution. Paraboloid domes combine high structural strength with reduced material usage, contributing to lightweight yet stable designs. From a mathematical perspective, the continuous curvature of a paraboloid simplifies stress analysis and load distribution modeling, further enhancing its suitability for engineering applications [4].

Similarly, the spherical cap, a portion of a sphere or hemisphere, is widely employed in real-world applications. Spherical caps are valued for their ability to distribute stress uniformly and resist internal and external pressure, which explains their extensive use in manufacturing, architectural structures, and chemical engineering systems for safety, strength, and design efficiency [5-7]. Pressure vessels, storage tanks, and dome roofs frequently utilize spherical-cap designs to optimize mechanical performance.

In order to effectively apply the concepts of paraboloids and spherical caps, visualization and quantitative understanding of their geometric properties are essential. Although many existing software tools can render the graphical shape of these surfaces, they often lack built-in functions for computing surface area and volume. Furthermore, users are typically required to manually define the mathematical equations governing these surfaces, creating a barrier for users without advanced programming or mathematical expertise.

Addressing these limitations, this research proposes PaSCap, an interactive MATLAB Graphical User Interface (GUI) that enables visualization and computation of key geometric parameters for both paraboloids and spherical-cap objects. The GUI requires minimal user input, the radius and height of the chosen geometry, before simultaneously displaying a three-dimensional graph along with automatically computed surface area and volume. By integrating analytical formulas with interactive visualization, PaSCap provides an accessible, educational, and practical computational platform for engineering design and conceptual analysis.

The remainder of this paper presents the mathematical formulations underlying paraboloid and spherical-cap geometries, followed by the design and implementation of the PaSCap GUI, illustrative examples demonstrating its capabilities, and a discussion concluding the study.

2. Methodology

In this study, the general dome, as illustrated in Figure 1, is considered with a height h and a circular base of radius r . The dome can be represented either as a paraboloid or as a spherical cap, depending on the geometric assumptions. This approach allows for flexible modeling of domed structures in architectural, engineering, and physical applications.

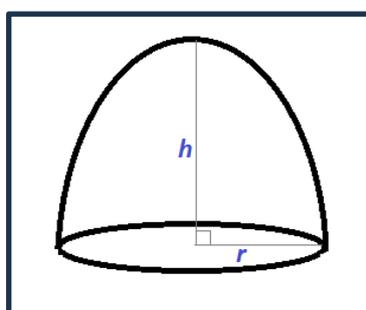


Fig. 1. A schematic diagram of a dome with height, h and radius, r

For the paraboloid representation, the dome is treated as a surface of revolution generated from a circular parabola. The corresponding solid G lies above the xy -plane and is bounded above by the paraboloid surface, which is described by the equation

$$z = -\frac{h}{r^2}(x^2 + y^2) + h. \quad (1)$$

The parameters r and h serve as the primary inputs for visualization and computation. The MATLAB GUI developed in this study, PaSCap, uses these inputs to render the 3D surface and compute key geometric properties. The volume V_p and surface area A_p of the paraboloid can be computed using the standard formulas for solids of revolution

$$V_p = \frac{1}{2}\pi r^2 h, \quad (2)$$

$$A_p = \pi r \sqrt{r^2 + 4h^2}. \quad (3)$$

These formulas can be derived by following the methods outlined in [8], and also described in [9].

For the spherical-cap representation, the dome is modelled as a portion of a sphere with radius R , determined by the height h and base radius r (see Figure 2). Moreover, the volume V_s and surface area A_s of the spherical cap are given by

$$V_s = \frac{1}{3}\pi h^2(3R - h), \quad (4)$$

$$A_s = 2\pi R h, \quad (5)$$

where $R = \frac{r^2 + h^2}{2h}$. The same GUI framework is used to generate the 3D visualization and compute these geometric parameters. Furthermore, in the GUI, the spherical cap (the blue surface in Figure 2) is shifted so that it also lies on the xy -plane, like the paraboloid. In this case, both the paraboloid and the spherical cap have circular bases on the plane $z = 0$.

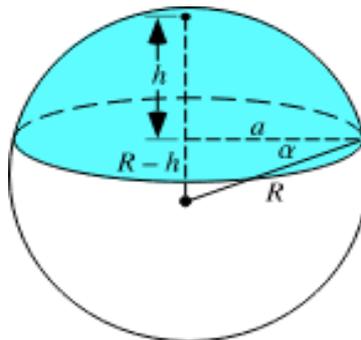


Fig. 2. The spherical cap of height h and base of radius a that is cut off from sphere of radius R

By integrating analytical formulas for both paraboloid and spherical-cap geometries with interactive visualization, the GUI, which is called as PaSCap, provides an accessible and practical computational platform for the analysis and design of dome structures.

3. The GUI

PaSCap (Pa is taken from **Paraboloid** while SCap is based on **Spherical-Cap**) is as an interactive MATLAB application that integrates user input, numerical computation, and three-dimensional visualization within a unified graphical interface. This study represents an extension of our previously developed GUI, which was limited to paraboloid geometry. By incorporating spherical cap geometry, the present work establishes clear novelty, whereas the earlier innovation is copyrighted under the Malaysian Intellectual Property Office (MyIPO) with reference number CRLY2024J05921. The software adopts an event-driven design in which user interactions, such as parameter entry or geometry selection, trigger computational routines and graphical updates. This framework for designing the GUI supports clarity and maintainability, which are essential characteristics of scientific and engineering software [10].

The operation of PaSCap follows a structured and sequential workflow. First, the user defines the geometric parameters and selects the dome type through the interface. Second, the software performs input validation to ensure that the provided geometric parameters are numerical, within feasible ranges, and logically consistent with the selected dome type. Third, the relevant analytical formulas are applied to compute the dome's surface area and volume. Finally, the numerical results and the corresponding visualization are presented simultaneously within the same interface. This workflow is summarized in the software workflow diagram shown in Figure 3, illustrating the logical flow between user input, computation, and visualization modules [11].

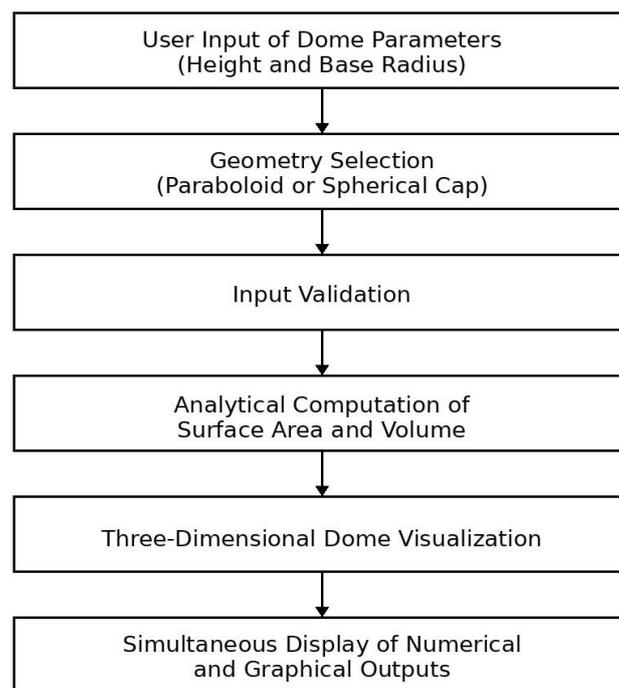


Fig. 3. The workflow of PaSCap as a MATLAB GUI, illustrates the sequence from user input of dome parameters (height and base radius), geometry selection (paraboloid or spherical cap), input validation, analytical computation of surface area and volume, three-dimensional dome visualization, and simultaneous display of numerical and graphical outputs within a single interface

The GUI is built using MATLAB's GUIDE framework and is designed so that only one instance of the program can run at a time. If PaSCap is already open, launching it again will not create a new window. This prevents conflicts with shared data or overlapping graphical elements, ensuring consistency throughout the session. Upon startup, the software automatically initializes the plotting area, assigns default parameter values, and prepares the necessary data structures for computation,

enabling immediate interaction. The overall interface layout, comprising input fields, geometry selection controls, numerical output displays, and visualization panels, is shown in Figure 4, based on the MATLAB .fig file of the developed GUI [10].

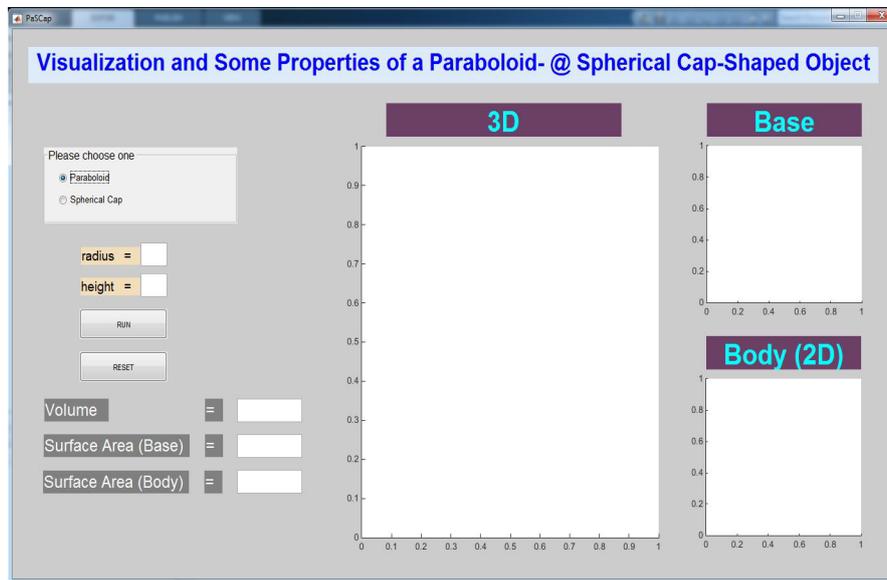


Fig. 4. Graphical user interface layout of PaSCap developed in MATLAB’s GUIDE, illustrating the arrangement of input parameters, geometry selection controls, numerical outputs, and three-dimensional visualization panels.

User interaction is centered on two geometric input parameters which are the dome height and the radius of the circular base. These parameters are entered through editable input fields, while selection controls allow the dome geometry to be specified as either a paraboloid or a spherical cap. Input validation procedures are incorporated to ensure that only feasible numerical values are processed, thereby improving computational reliability and robustness [12].

Following successful validation, the computational module applies the analytical expressions presented in the Methodology section to evaluate the surface area and volume of the selected dome. These computational routines are implemented independently of the graphical interface, enhancing code readability and facilitating future modification or extension of the software [12]. In addition to numerical computation, the visualization module generates both two-dimensional and three-dimensional graphical representations of the dome using MATLAB’s built-in plotting tools. The two-dimensional plots illustrate the base profile and cross-sectional shape of the dome, while the three-dimensional plot presents the full dome geometry. All visual outputs update automatically in response to changes in input parameters or geometry selection, with interactive features such as rotation and zooming supporting improved spatial understanding [11].

Overall, PaSCap emphasizes usability, modular organization, and effective integration of computation and visualization. By combining analytical modeling with an interactive graphical layout, the software provides a practical tool for educational applications, preliminary engineering analysis, and conceptual exploration of dome geometries. Similar interface-driven software designs have been reported to improve learning outcomes and user engagement in MATLAB-based engineering applications [7,13].

4. Geometric Simulation and Visualization Using PaSCap

As a representative example of a real architectural dome, the internal dome of Palau Güell is examined in this study. The dome forms the primary vertical spatial element of the interior hall and is situated beneath the rooftop lantern and chimney structures. Prior geometric studies indicate that the Palau Güell dome is more accurately described by a general quadric surface rather than an ideal paraboloid [4]. For illustrative purposes, the dome is modeled as a rotational paraboloid in PaSCap. Based on reported reconstructed geometric data, the characteristic dome height is approximated as $h = 16$ m, with an effective base radius of $r = 4.5$ m. These values represent representative dimensions of the interior dome rather than exact parameters of the original surface, allowing rapid visualization and exploration of its overall form and spatial characteristics.

Using these parameters, PaSCap generates both two-dimensional and three-dimensional visualizations of the dome geometry (see Figure 5), providing a practical framework for examining how an idealized paraboloid approximation affects overall form and spatial interpretation. PaSCap also automatically computes key geometric properties, such as surface area and volume, offering quantitative insight that is not typically available in traditional descriptive or graphical studies. This example highlights the usefulness of PaSCap as a rapid visualization and exploratory analysis tool for preliminary architectural investigations, conceptual geometry studies, and educational applications involving complex dome structures [7,11-13].

As a contrasting example of a spherical-cap geometry, a traditional Inuit igloo is modeled. Igloos are commonly represented as hemispherical or near-hemispherical caps due to their rotational symmetry and efficient stress distribution [14]. Typical adult-sized igloos have an internal base radius of approximately 2.0–2.5 m and a corresponding height of about 1.8–2.2 m. For the purposes of modeling in PaSCap, a representative base radius of 2.2 m and a height of 2.0 m are adopted, ensuring a geometrically valid spherical cap. In this idealized model, the igloo is treated as a complete spherical cap without entrance tunnels or doors, which are commonly omitted in geometric analyses for simplicity (see Figure 6). Modeling an igloo with these parameters illustrates PaSCap's suitability for visualizing small-scale spherical-cap structures and highlights its potential in educational contexts for exploring traditional architecture and understanding basic structural geometry.

It is important to note that the original studies of the Palau Güell dome and traditional igloos primarily focused on geometric form, construction techniques, or spatial characteristics and did not report surface area or volume measurements. PaSCap addresses this limitation by automatically computing these geometric properties based on the specified input parameters, thereby providing quantitative information that complements the visualizations. Overall, PaSCap enables users not only to visualize dome geometries but also to assess their spatial characteristics in a unified framework, supporting preliminary design analysis, comparative studies, and educational exploration. By integrating visualization with quantitative evaluation, PaSCap offers a clear advantage over traditional descriptive or purely graphical approaches, facilitating rapid insight into complex dome geometries and their geometric implications.

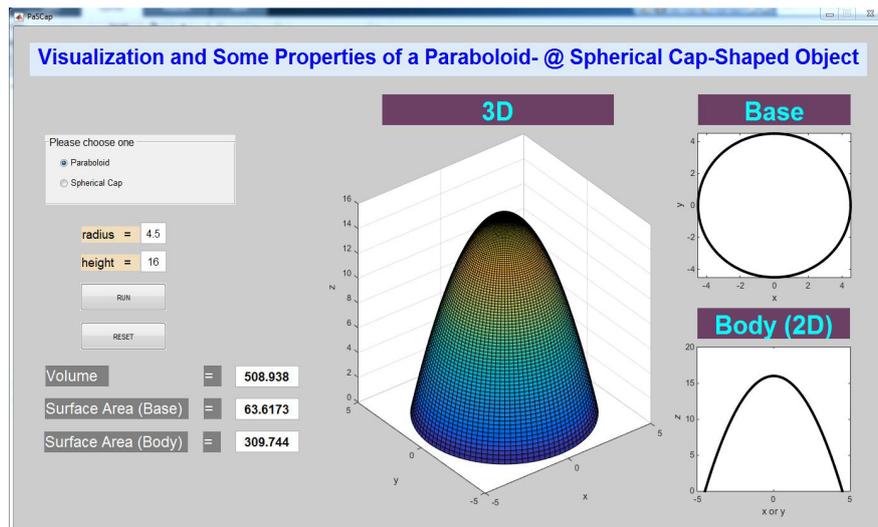


Fig. 5. Outputs of PaSCap when inputs are $h = 16$ and $r = 4.5$ for visualization and estimating the volume as well as surface area of Pulau Güell

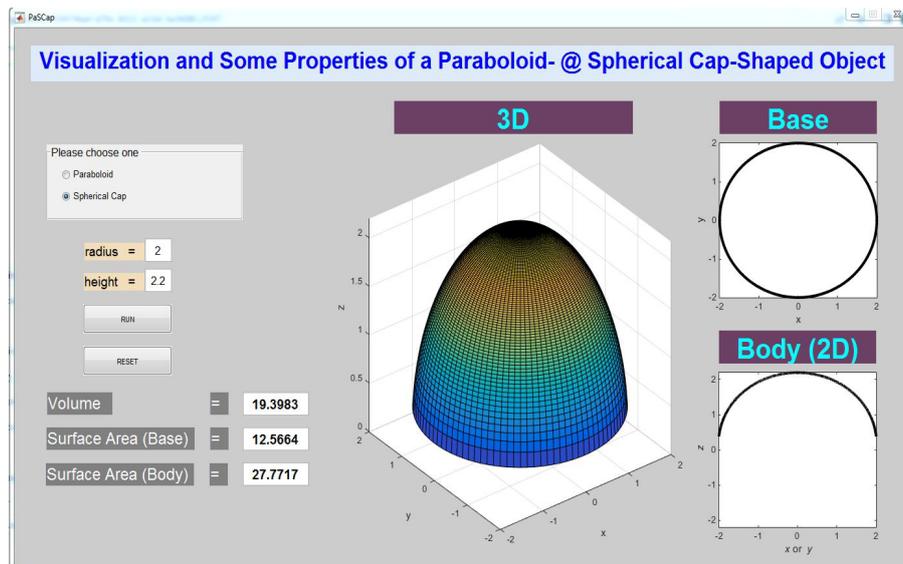


Fig. 6. Outputs of PaSCap when inputs are $h = 2$ and $r = 2.2$ for visualization and estimating the volume as well as surface area of an igloo

Next, PaSCap is applied to model a historical charcoal kiln on Langkawi Island, Malaysia, which, despite its deteriorated condition, is already utilized as a heritage attraction for tourism and edutourism. Traditional charcoal kilns were typically built with dome-like shapes to enhance structural stability, heat retention, and airflow during the carbonization process. This study now investigates how idealized geometry influences the kiln's physical characteristics by modeling it as both a paraboloid and a hemispherical (spherical-cap) form, each with the same circular base radius and a height of 3 m. Parameters are entered into PaSCap to generate two-dimensional cross-sectional plots, three-dimensional visualizations, and numerical outputs of surface area and enclosed volume. The kiln door is excluded from the model, as it is closed once the wood is loaded and the burning process begins.

Figure 7 and Figure 8 respectively show the results for the paraboloid and spherical cap shaped charcoal kiln models. The numerical results also reveal clear geometric differences between the two models. The paraboloid produces a curved surface area of 47.9737 m² and a total surface area of 42.4115 m², while the hemispherical model yields a larger curved surface area of 56.5487 m², which is also equal to its enclosed volume for the given dimensions. In both cases, the base surface area remains constant at 28.2743 m², reflecting the identical circular footprint of the kiln. Note that the results in Figure 7 are the same as those in Khairuddin *et al.*, [9], while another study that models the shape of the charcoal kiln can be found in Ahmad Khairuddin *et al.*, [15].

The kiln body's surface area determines the quantity of construction material required, such as clay, laterite, or stone, enabling planners to estimate material needs and costs when rebuilding the structure. Accurate surface area estimates also guide decisions regarding the extent of exterior finishes or protective treatments needed to enhance durability while maintaining the kiln's original appearance. Evaluating the enclosed volume provides insights into operational aspects, including firing duration, fuel efficiency, functional capacity, the amount of wood processed, and internal airflow affecting heat retention. These quantitative insights allow reconstructed kilns to reflect both the visual form and historical function of the original structure.

By providing detailed calculations of surface area and enclosed volume, PaSCap supports accurate digital modeling of different dome geometries, facilitating informed reconstruction and interpretation. Comparing paraboloid and hemispherical forms using these measurements enables design decisions even when physical remains are incomplete or degraded. In addition, PaSCap allows stakeholders to explore multiple reconstruction scenarios digitally before any physical rebuilding occurs, reducing uncertainty and preserving the kiln's authenticity. Consequently, PaSCap serves as a powerful tool for enhancing cultural tourism and edutourism by integrating mathematical modeling, visual representation, and heritage-based learning within a single interactive platform.

5. Further Discussion and Conclusion

While PaSCap has proven valuable for modeling dome structures and supporting tourism, its potential extends far beyond these applications. Its core capabilities of visualization, surface area, and volume computations enable detailed design analysis and transform abstract concepts into tangible insights. Such quantitative geometric modeling is widely recognized as essential in engineering and architectural analysis, where accurate surface area and volume calculations directly inform material estimation, performance evaluation, and design optimization [16].

In education, PaSCap helps students and researchers understand complex structural details. By simulating surface area and volume, users can quantify material requirements, examine proportions, and assess spatial efficiency [17]. Visualization and spatial reasoning are critical for success in STEM fields, as they allow learners to interpret and manipulate three-dimensional data fundamental to engineering and design practice [18]. Together, interactive visualization and quantitative analysis promote deeper comprehension and hands-on engagement with complex spatial concepts.

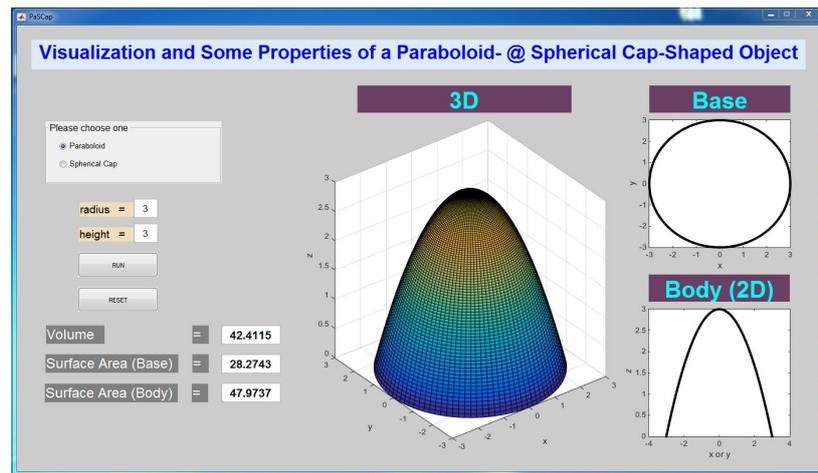


Fig. 7. Outputs of PaSCap with inputs $h = r = 3$ for visualization, volume estimation, and surface area estimation of a paraboloid charcoal kiln model

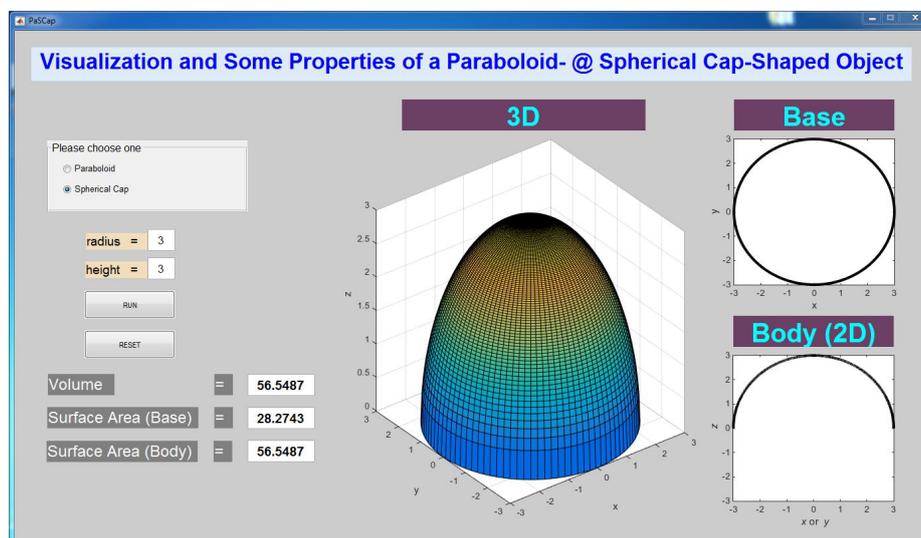


Fig. 8. Outputs of PaSCap with inputs $h = r = 3$ for visualization, volume estimation, and surface area estimation of a spherical cap charcoal kiln model

Moreover, PaSCap's ability to calculate surface area and volume also provides practical advantages in industry. Engineers and designers can assess material consumption, optimize dimensions, and predict structural behavior before construction [19]. By simulating designs virtually, PaSCap reduces errors, streamlines resource allocation, and allows rapid iteration. Furthermore, its visualization outputs can be integrated with virtual reality (VR) and augmented reality (AR) platforms, enabling immersive exploration of models, improved spatial understanding, and more effective design evaluation and decision-making processes [20].

PaSCap also supports environmental and structural analysis. Surface area calculations inform heat transfer and airflow studies, while volume computations assist in capacity planning and spatial optimization [16]. These capabilities enable designers and engineers to evaluate performance, material efficiency, and operational feasibility without relying solely on physical prototypes, thereby reducing cost, time, and resource consumption.

Therefore, PaSCap demonstrates significant value through its visualization capabilities and accurate computation of surface area and volume. By supporting precise geometric modeling and

enhancing spatial understanding, it contributes to informed decision-making and serves as a powerful resource for education, industry, and applied engineering analysis.

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