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Validation of the NERAPH Model for Cannabis Consumption through Sensitivity Analysis

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ABSTRACT

Cannabis, an illicit narcotic, poses a significant public health risk in developing nations due to its multiple negative health consequences. The transmission of cannabis use occurs through interaction between users and non-users. This manuscript's introduces a novel aspect in the extending form the Non-User, Experimental Users, Recreational Users, Addicts, and Hospitalised Individuals (NERAH) model for cannabis usage, by integrating a new class of prisoners. This addition is crucial as it enhances the realism of the model, addressing the challenge of accurately representing cannabis use patterns. The real-world problem is mathematically represented using first-order nonlinear ordinary differential equations, resulting in a modified mathematical model for cannabis consumption. The general population is categorized into two major groups: cannabis users and non-users. Cannabis users are further divided into five distinct classes, each representing a specific stage or level of substance use.The invariant region and basic reproduction number are key components of the methodology, used to validate the updated model and determine the initial transmission rate of marijuana use in the general population. Sensitivity analysis was conducted to identify critical factors contributing to cannabis growth. Based on these findings, control strategies were proposed to prevent cannabis usage. Numerical simulations were performed using the fourth-order Runge-Kutta method in MATLAB, effectively demonstrating the revised model's ability to enhance understanding and efficiency. Both the methodological and numerical results support the proposed model, providing valuable insights for cannabis control strategies.

1. Introduction

The growth of a nation faces obstacles not just by crime and conflict, nevertheless by illicit drug use among its citizens. Any pharmacological ingredient utilised for speeding up any kind of chemical

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reaction in a person's body, such as relieving an illness or improving mental function, is referred to as a medication. Among the various substance kinds, cannabis is the one that the study I am conducting is mostly focused on. Many people believe that cannabis is a "gateway" drug for more potent substances, or that using cannabis is a symptom of using other illegal substances. However, kids who use the gateway substances are more likely to advance to other, more dangerous drugs [1]. One of the most commonly misused illicit substances in the globe is cannabis, it is also know by various names as shown in Figure 1.

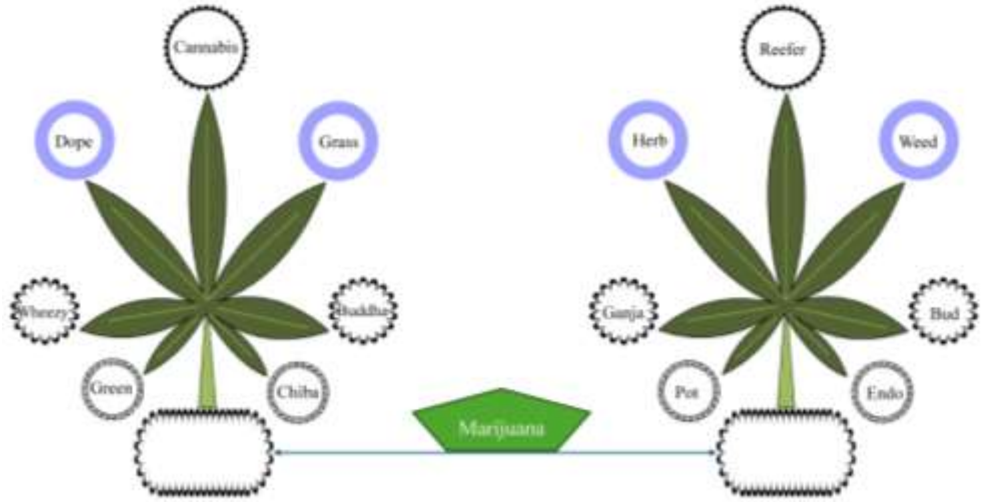

Fig. 1. Various names of cannabis

A rising trend in cannabis use reporting prevalence in medical centres, with certain large cities reporting rises of up to 121 percent from the years 1999 and 2000. Additionally, cannabis accounted for 25 percent of illicit drug use treatment hospitalisations in the year 1997, increasing since 1992. Nevertheless, laws regarding medical marijuana (MML) have been implemented in 29 states in the US and the District of Columbia [2], and legalised consumption of cannabis for recreational purposes in eight different states. Introducing state laws that allow cannabis consumption could potentially make use of cannabis more likely. The use of cannabis is linked to a higher risk of taking other drugs, which include cigarettes. These research look at the eight times higher risk of cigarette start linked to smoking cannabis. Similarly, smoking cigarettes may make consuming cannabis more likely [3].

State-level legalisation for both medicinal and leisure activities has grown in the past decade. Eleven states and the District of Columbia have legalised cannabis for purposes of enjoyment, and more states are now considering implementing similar laws. As of July 2019, medical cannabis was allowed in 33 states. There have been recent indications of a rise in cannabis-related negative health impacts due to rising cannabis consumption habits and effectiveness [4]. Nevertheless, the majority of these correlations have been derived from observational research, reports of cases, or scenario studies [5].

The sativa variety of cannabis is an agricultural plant whose flowers, leaves, and seeds as seen in Figure 2 are dried and compacted into a green and brown mixture known as marijuana. It is often consumed in pipe or hand-rolled cigarette. Certain individuals, nonetheless, prepare it as tea or combine it into meals. It's crucial to remember that people who use of cannabis begin to feel its effects nearly instantly after smoking it, and these effects last for up to three hours. On the opposing hand, the effects of cannabis remain deeper while consumed in meals. Cannabis primary addictive ingredient is delta-9-tetrahydrocannabinol (THC). Cannabidiol receptors for antibodies, which are present on neurones throughout the nervous system, are the site of action for THC.

Fig. 2. Cannabis in the form of flowers, dried leaves, and seeds

Particularly, areas of the brain that affect happiness, recall, cognitive focus, perception of sensation, and controlled movement contain receptors for cannabinoid substances. In addition to increasing the chances of lung infections, smoking cannabis over quadruples the chance of heart attacks during the first hour after consumption. Even more concerning, a big potential study revealed that individuals who start using cannabis as teens suffer a substantial reduction in their overall IQ, as well as that the intellectual talents they lost could never recovered when they finally stopped consuming cannabis as grownups [6]. Additionally, there are more cases of lung diseases and breathing problems, indicating that vaping poses a growing risk to a person's health [7]. Individuals are becoming more curious regarding the heart risk associated with cannabis as its usage grows, particularly when combined with other frequently recommended cardio medications. However, there are now insufficient rigorous academic research and guidelines to direct the heart failure. disease belonging, and the impact of cannabis on the heart remain unclear [8].

Because of its complicated molecular composition and the psychoactive compounds cannabis is also utilised as medicine. The positive impact associated with cannabis is still not effectively determined because of the greater number of over 200 compounds which are currently found and unidentified ones. To obtain the cannabis compounds, flower of cannabis are either boiled or vaporised [9]. A significant amount of cannabis consumption in any community can have a number of negative effects, including a rise in criminal activity and a consequent rise within property and life losses, additional fatalities from injuries and offences, an apparent decline in the number of effective man-hours, a decrease in the possible benefit of individuals to the economic growth caused by cannabis consumption, along with ongoing, massive costs for preventive measures as well as the medical treatment of addicted consumers [10].

The brains of people who use cannabis are altered both during the short-term and over the long term. Whenever an individual consumes cannabis, the psychoactive ingredient THC is rapidly transported through the respiratory system into the circulatory system. This is an immediate effect. The substance gets circulated across the human organism via the bloodstream, including to the cognitive system and other tissues. It takes longer for the human body to process THC once it is consumed in the form of meals or beverages. For this instance, patients often begin to experience the consequences thirty minutes to one hour after the occurrence. An excessive amount of activation of the region of the human brain that have the greatest number of sensors is caused by cannabis.

Long-term consequences of cannabis include an impact on the growth of the central nervous system. Some people who begin smoking cannabis while they are adolescents may experience impairments in their cognitive, mental, and academic abilities, as well as changes in the way their minds constructs associations within the regions that are important for these kinds of activities. The duration of the adverse effects of cannabis and the possibility that some of the alterations may be permanent remain under consideration by academics [11].

Cannabis use poses a significant threat to public health [12]. In the United States, cannabis was an extremely frequently consumed illicit drug. Recent studies have shown that 18.7 million Americans have admitted to consuming drugs, with 75% of cannabis consumers reporting persistent usage [13]. More than seventy-two million Americans have additionally tried cannabis.

The global burden of mortality and morbidity associated with illicit drug (Cannabis) use constitutes a substantial public health challenge for developed as well as developing nations. Thus, an effective approach to examining this societal widespread is to determine and predict its dynamic advancement concerning individuals [14]. In 2010, it was estimated that approximately 5% of the global adult population, equating to 230 million individuals, had used a prohibited substance at least once. Prohibited usage of cannabis continues to impose a considerable burden, resulting in the loss of significant life expectancy and successful careers for many individuals [14]. In 2012, approximately 183,000 fatalities associated with drugs were documented [15]. It was determined that among 162 million and 324 million individuals aged 15 to 64 worldwide had consumed an illegal substance. In 2014, deaths caused by drugs were recorded at approximately 207,400 for individuals aged 15-64 globally [16]. Cannabis use constitutes a multifaceted social and health issue that impacts numerous families and communities [17].

On the basis of written documentation through past biological history and conventional literature on herbal treatments, cannabis has been proven documented to exhibit particular therapeutic characteristics. Throughout history, cannabis has been utilised for the treatment of a variety of medical ailments, including persistent inflammation and seizures [18]. The consumption of cannabis, on the opposing hand, is considerably more frequently linked to engaging in recreational activities, criminal activity, including societal concerns. Having the reason of this analysis, the word "medical cannabis" is utilised when referring to the exploitation of cannabis for solely the purpose of providing medical assistance [19].

The scenario quite similar to this one exists in Thailand, although the possession of cannabis for medical treatment is permitted with a written prescription by an authorised Thai-traditional healthcare professional, a physician, or a dental professional. In the state of Selangor, Malaysia, the purpose of this research is to investigate the general level of medical cannabis consumption amongst adults and the characteristics that are connected with their approval of the decriminalisation of cannabis for medicinal purposes. In a manner that is analogous to that which can be learnt from countries such as Germany, Canada, and Thailand as well, the findings may be utilised to devise appropriate measures that strike an equilibrium within the implementation of strategic interventions and the possible growth of healthcare facilities in the context of providing impacted individuals with the opportunity for medicinal cannabis therapy [20].

The total number of individuals in Malaysia who are addicted to drugs and other substances has increased by eight percent between the years 2018 and 2019, according to the latest data [21,22]. in comparison with other Malaysian states, Selangor encompasses the smallest percentage of illicit drugs offenders and addicts to drugs per speculated overall population. This result might be attributed to the fact that people who live in cities suffer from an increased degree of understanding of concerns pertaining to addiction, as well as the potential hazards that are associated with these problems [23,24]. Users that met the requirements for being included for the research were

Malaysians who were at least 18 years old and lived in the state of Selangor throughout the time period of the research [25]. When it comes to medical cannabis, despite the fact there continues to be a lack of research evidence about its therapeutic capabilities in the treatment of a variety of conditions, the assertions of its advantages are extensively disseminated, debated, and pushed within the general population.

Cannabis users come in a variety of forms, including experimental, recreational, and addicted users. Individuals who supply themselves for the sake of an investigation are known as experimental users. For an indeterminate time, experimental users take cannabis as a test drug. The primary cause is ignorance regarding cannabis consumption and dependence. Frequent recreational cannabis users exercise oversight of the kind and dosage of cannabis they consume because they use it in a prescribed way. They are in charge of deciding how much and where to utilise it. They typically have an option in the way they use cannabis, and they generally consume it in social settings. Additionally, certain cannabis users have been shown to be hooked and utilise it as a routine. These people develop drug dependence as a result of their regular and ongoing consumption of cannabis. The person who is addicted needs cannabis for emotional or physically concerns. Without the cannabis, they are unable to confront the outside world. They appear to desire to get away from everyday life. Other users considered a novel and fascinating way of living and how they should approach it, while the situation with addicted is extremely different. Additionally, addicts want to use cannabis often and constantly; they are not concerned about the amount of cannabis that they consume since getting it becomes more essential to them than its purity [26].

Mathematically, Ullah *et al.,* [27] described the characteristics of illegal cannabis use in the general population. This included those who were using cannabis and those who were not. Four more categories experimental (E_S), recreational (R_S), addicted (A_S), and hospitalised (H_S) were added to better categorise cannabis users as shown in Eq. (1).

$$
\begin{aligned}
dN_S(t)/_{dt} &= £ - (cR_S(t) + c_3A_S(t) + \mu)N_S(t) + c_2E_S(t) + c_4R_S(t) + c_7H_S(t),\\
dE_S(t)/_{dt} &= cR_S(t)N_S(t) - (c_1 + c_2 + \mu)E_S(t),\\
dR_S(t)/_{dt} &= c_3A_S(t)N_S(t) + c_1E_S(t) - (c_4 + c_5 + \mu)R_S(t),\\
dA_S(t)/_{dt} &= c_5R_S(t) - (c_6 + \mu)A_S(t),\\
dH_S(t)/_{dt} &= c_6A_S(t) - (c_7 + \mu)H_S(t).
\end{aligned} \tag{1}
$$

Subsequently, we then modified the system of equations as seen in Eq. (1) by incorporating a new class of prisoners, individuals to become a new model known as Non-users, Experimental users, Recreational users, Addicted, Hospitalised individuals, and Prisoners (NERAPH) model. We also validated the NERAPH model, which establishes a connection between cannabis consumption and demographic characteristics. Numerous researchers have examined consumption of cannabis and developed mathematical models aimed at reducing and preventing cannabis use within the population; however, they have neglected the significant demographic of prisoners in their models, providing initiatives to alleviate and prevent cannabis smoking considerably challenging.

2. Formulation of the Modified Mathematical Model

Mathematical models have been considered to be one of the most popular types of epidemiological frameworks used in empirical and experimental studies. Although in only a handful of particular phases, people within these representations display an extensive variety of features;

several of such circumstances only have declarations which explain the individuals' distinctive features, which could be epidemiologically essential to an evolving framework, such as a harmful agent or the host itself. Computational methods for solving "non-linear ordinary differential equations" including both local as well as nonlocal components have been acknowledged as a valuable mathematical technique [28]. For a more comprehensive overview of the mathematical models of drugs consumption, read [29-47].

Considering the perspective of a global spread, we are currently creating an improved version of the existing model [27], by incorporating a new class of prisoners to become an updated (NERAPH) model as discussed in the second last section of introduction of this article. The general population can be divided into two main groups: marijuana users and non-users. Smokers fall into five different types, each of which corresponds to a different stage of the dependency process. These phases are included in Table 1 below, along with susceptible people. We used the data, in the form of mathematical values for the parameters, from the previous literature, as shown in Table 2 of this manuscript. The current manuscript describes the methodology for validating the suggested mathematical model for enhanced performance. In the methodology section, the invariant region and basic reproduction number serve as crucial components for validating the updated model. The invariant region aids in determining the boundedness and positivity of the model, while the basic reproduction number determines the initial transmission rate of cannabis. Eq. (2) displays the entire population T_p(t) at a time 't' and Figure 3 represents the schematic representation of the updated model [33].

$$T_p(t) = C_{nu}(t) + C_{eu}(t) + C_{ru}(t) + C_{au}(t) + C_{hu}(t) + C_{pu}(t). \quad (2)$$

The subsequent set illustrates the standard conduct of cannabis consumers (as indicated by Eq. (3)) in "first-order non-linear ordinary differential equations." And schematic representation of the NERAPH model are given in Figure 3.

Table 1
Comprehension of State Variables

State variables	Comprehension
C_{nu}	Cannabis: Non-users
C_{eu}	Cannabis: Experimental users
C_{ru}	Cannabis: Recreational users
C_{au}	Cannabis: Addict’s class
C_{hu}	Cannabis: Hospitalized individuals
C_{pu}	Cannabis: Prisoner’s class

$$\begin{aligned}
\frac{dC_{nu}(t)}{dt} &= \mathcal{M} - s_1 C_{ru}(t) C_{nu}(t) - s_4 C_{au}(t) C_{nu}(t) - \mu C_{nu}(t) + s_3 C_{eu}(t) + s_5 C_{ru}(t) + s_9 C_{hu}(t) \\
&\quad + s_{10}(1-\alpha_1) C_{pu}(t), \\
\frac{dC_{eu}(t)}{dt} &= s_1 C_{ru}(t) C_{nu}(\mathrm{t}) - (s_2 + s_3 + \mu) C_{eu}(t), \\
\frac{dC_{ru}(t)}{dt} &= s_4 C_{au}(t) C_{nu}(\mathrm{t}) + s_2 C_{eu}(\mathrm{t}) - (s_5 + s_6 + \mu) C_{ru}(t), \\
\frac{dC_{au}(t)}{dt} &= s_6 C_{ru}(\mathrm{t}) + s_{10}\alpha_1 C_{pu}(\mathrm{t}) - (s_7 + s_9 + e + \mu) C_{au}(t), \\
\frac{dC_{hu}(t)}{dt} &= s_7 C_{au}(\mathrm{t}) - (s_8 + \mu) C_{hu}(t),
\end{aligned} \quad (3)$$

$$\frac{dC_{pu}(t)}{dt} = s_9 C_{au}(t) - (s_{10} + \mu) C_{pu}(t).$$

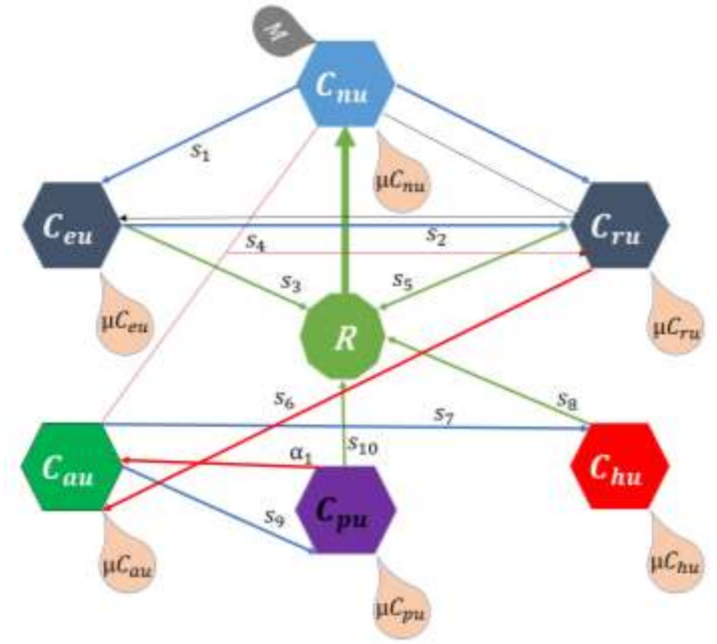

Fig. 3. Schematic illustration of the modified model

Table 2 contains complete explanations of the proposed updated model parameters.

Table 2
Explanations of the parameters

Notations	Parameters Explanations	Values	References
$\mathcal{M}$	The population's birth rate	$0.0015875 day^{-1}$	[48]
s_1	Influence of experimental participants on smoke-free individuals (at risk)	$0.446 day^{-1}$	[36]
s_2	The ratio of individuals engaging in experiments transmit to those using substances recreationally	$0.5 day^{-1}$	[36]
s_3	Addicts' rate of influence over susceptible	$0.001201 day^{-1}$	[33]
s_4	How many people in the study were able to quit smoking once receiving guidance	$0.17 day^{-1}$	[33]
s_5	Likelihood that recreational cannabis consumers might stop using according to their hectic surroundings	$0.002 day^{-1}$	[33]
s_6	How frequently accomplish people who use drugs for fun become dependent after the adjustment time is over?	$0.025 day^{-1}$	[33]
s_7	How many people who take drugs regularly end up in prison	$0.22 day^{-1}$	[36]
s_8	The percentage of convicts who served out their treatment phase	$0.2010 day^{-1}$	[33]
s_9	Substance abuser prison time rate	$0.0157871 day^{-1}$	[32]
s_{10}	How many prisoners actually receive treatment for their problems ?	$0.0331 day^{-1}$	[36]
μ	Natural mortality ratio among populations	$0.006 day^{-1}$	[36]
$\mathbf{e}$	The number of deaths that police departments encounter	$0.0005 day^{-1}$	[33]
α_1	How many offenders return to the addiction wing after serving their prison time	$0.03 day^{-1}$	[33]

2.1 Invariant Region

We presumed that all state variables and features (parameters) of the suggested framework are positive at time t = 0, as they are relevant to the real-world community within which we exist. This encompasses the behaviour of the whole humanity, as illustrated by the non-linear system below [33].

The overall population is outlined in the Eq. (2) provided previously. Eq. (2) is solved to derive the subsequent equation.

$$\dot{T_p}(\mathrm{t}) = \mathcal{M} - \mu T_p(t) - eC_{au}(t), \tag{4}$$

Eq. (4) gives us

$$\dot{T_p}(\mathrm{t}) = \mathcal{M} \leq \mu T_p(t), \tag{5}$$

Upon solving Eq. (5), we conclude

$$\dot{T_p}(\mathrm{t}) \leq T_p(t)(0)e^{-\mu\mathrm{t}} + \frac{\mathcal{M}}{\mu}(1 - e^{-\mu t}) \Rightarrow \ T_p(\mathrm{t}) \leq \frac{\mathcal{M}}{\mu} \text{ when t} \dashrightarrow \infty \tag{6}$$

Taking into account the previous assessment, we generated a subsequent proposition.

2.2 Proposition

The boundaries of the proposed modified model is specified by

$$\mathcal{P} = \left[\left(C_{nu}(t) + C_{eu}(t) + C_{ru}(t) + C_{au}(t) + C_{hu}(t) + +C_{pu}(t)\right) \epsilon\, \mathcal{R}_+^6, T_p(\mathrm{t}) \leq \frac{\mathcal{M}}{\mu}\right] \text{[33]}. \tag{7}$$

The domain exhibits positive invariance, the modified model is rigorously developed both computationally and mathematically, and all system boundaries indicate the trajectory of migration. This suggests the existence of a population threshold. Furthermore this represents the preliminary phase in establishing the validity of the formulated model [48].

3. Reproductive Number

A basic epidemiological measure that indicates the average quantity of insignificant addictions produced by simply one addict in a community that does not smoke is the reproduction number, or $\mathcal{R}_0$. It is determined by a standard of transmission and the time frame of dependency and is an important indicator of the likelihood for substance or illness dissemination. Substance effectiveness is probably going to continue transmission and cause a nationwide outbreak if the $\mathcal{R}_0$ value is more than 1, nevertheless a figure less than 1 implies that individuals will eventually lose the effectiveness of the substance. A well-known technique called the "Next Generation Matrix Method" is used to calculate the initial rate of networks (reproduction number) [49,50].

$$\text{As} \quad \delta(\mathbb{F}\mathbb{V}^{-1}) = \mathcal{R}_0 \text{ [48, 51]}. \tag{8}$$

where the spectrum is specified by δ. Moreover, $\mathbb{F} = \mathfrak{J}_{\mathbb{f}}$ represents the Jacobian of $'\mathbb{f}'$

$$\mathbb{F} = \begin{pmatrix} \mathbb{f}_1 \\ \mathbb{f}_2 \\ \mathbb{f}_3 \end{pmatrix} = \begin{pmatrix} s_1 C_{ru} C_{nu} \\ s_3 C_{au} C_{nu} \\ 0 \end{pmatrix}. \tag{9}$$

The full column in Eq. (9) represents those participants that develop addiction.

$$\mathbb{F} = \begin{pmatrix} \mathbb{F}_{11} & \mathbb{F}_{12} & \mathbb{F}_{13} \\ \mathbb{F}_{21} & \mathbb{F}_{22} & \mathbb{F}_{23} \\ \mathbb{F}_{31} & \mathbb{F}_{32} & \mathbb{F}_{33} \end{pmatrix} = \begin{pmatrix} 0 & s_1 C_{nu} & 0 \\ 0 & 0 & s_3 C_{nu} \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

For conciseness, we persued Eq. 10 as

$$\mathbb{F} = \begin{pmatrix} 0 & \mathfrak{p}_1 & 0 \\ 0 & 0 & \mathfrak{p}_2 \\ 0 & 0 & 0 \end{pmatrix}_{(MFE)}. \tag{11}$$

where $\mathfrak{p}_1 = \mathrm{s}_1 C_{nu}$ and $\mathfrak{p}_2 = s_3 C_{nu}$.

In a similar manner, $\mathbb{V} = \mathfrak{J}_{\mathbb{v}}$ represents the Jacobian of $'\mathbb{v}'$ where

$$\mathbb{V} = \begin{pmatrix} \mathbb{v}_1 \\ \mathbb{v}_2 \\ \mathbb{v}_3 \end{pmatrix} = \begin{pmatrix} -(s_2 + s_4 + \mu) C_{eu} \\ s_2 C_{eu} - (s_6 + s_5 + \mu) C_{ru} \\ s_6 C_{ru} - (s_7 + e + \mu) C_{au} \end{pmatrix}. \tag{12}$$

The people who join or leave the affected class are shown in the column of the matrix (Eq. (12)), beyond the people who belong to the most affected (non-smookers) group.

$$\mathbb{V} = \begin{pmatrix} \mathbb{V}_{11} & \mathbb{V}_{12} & \mathbb{V}_{13} \\ \mathbb{V}_{21} & \mathbb{V}_{22} & \mathbb{V}_{23} \\ \mathbb{V}_{31} & \mathbb{V}_{32} & \mathbb{V}_{33} \end{pmatrix}, \tag{13}$$

$$\mathbb{V} = \begin{pmatrix} -(s_2 + s_4 + \mu) & 0 & 0 \\ s_2 & -(s_6 + s_5 + \mu) & 0 \\ 0 & s_6 & -(s_7 + e + \mu) \end{pmatrix}_{(MFE)}. \tag{14}$$

For better understanding, we discovered Eq. (14) as

$$\mathbb{V} = \begin{pmatrix} -\mathfrak{h}_1 & 0 & 0 \\ c_1 & -\mathfrak{h}_2 & 0 \\ 0 & 0 & -\mathfrak{h}_3 \end{pmatrix}_{(MFE)}. \tag{15}$$

where $\mathfrak{h}_1 = (s_2 + s_4 + \mu)$, $\mathfrak{h}_2 = (s_5 + s_6 + \mu)$, and $\mathfrak{h}_3 = (s_7 + e + \mu)$.

The most important eigenvalue of $(\mathbb{F}\mathbb{V}^{-1})$ is:

$$\sqrt{\left(\frac{s_1 * \mathcal{M} * s_2}{\mathfrak{h}_1 * \mathfrak{h}_2}\right)}, \tag{16}$$

which is the fundamental formula that represents the result of the reproduction number

$$\mathcal{R}_0 = \sqrt{\frac{s_1 s_2 \mathcal{M}}{(s_2+s_4+\mu)(s_5+s_6+\mu)\mu}}. \tag{17}$$

4. Sensitivity Analysis of the Updated Model

The computational results for the sensitivity indexes was determined using the parameter values listed in Table 2. Table 3 displays the parameters of the mathematical framework along with their corresponding sensitivity indices. Model settings with sensitivity index values close to −1 or +1 indicate that alterations in their orders of magnitude significantly affect the increase or decrease of the reproduction number $\mathcal{R}_0$. The data presented indicate that s_1 exhibits the highest sensitivity in relation to $\mathcal{R}_0$. An increase of 10% in s_1 will result in a corresponding increase of 10% in $\mathcal{R}_0$. Therefore, it is essential to develop effective strategies that focus on diminishing the associations between vulnerable individuals and users of illicit substances in order to reduce the prevalence of consumption of illicit drugs and its negative impacts on the general population [52].

The suggested system, a first-order non-linear ordinary differential equation model pertaining to human population dynamics, may modify its outcomes with any alteration in parameter values. We can determine the most sensitive factors using sensitivity analysis in MATLAB and thereafter concentrate on them for future methodological studies. This paragraph emphasizes the crucial role of sensitivity analysis in mathematical modelling of real-world problem phenomena, as it requires the use of the most sensitive parameters to achieve optimal control variables, which are necessary for optimal control techniques in future studies.

The normalised progressive sensitivity values of an attribute concerning a parameter denote the percentage in terms of the absolute advancement of the variable to the associated variation in the parameter [33].

4.1 Definition

The generalised forward-sensitive values of $\mathcal{R}_0$ are dependent differentiably on any parameter ꝑ, of the system (3) is taken into account. These processes are characterised as

$$\text{Ꝑ}_{\text{ꝑ}}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \text{ꝑ}} \times \frac{\text{ꝑ}}{\mathcal{R}_0}. \tag{18}$$

Table 3 demonstrates the key parameters and their sensitivity values.

Table 3
Indexes of how sensitive the parameters are

Key parameters	Values	Sensitivity indexes
$\mathcal{M}$	0.0015875	+0.5000
s_1	0.446	+0.5000
s_2	0.5	+0.1309
s_4	0.17	−0.1258
s_5	0.002	−0.0303
s_6	0.025	−0.3788
μ	0.006	−0.5954

Figure 4 represents the visual illustration of the sensitivity indices of the proposed updated model key parameters as described in Table 3.

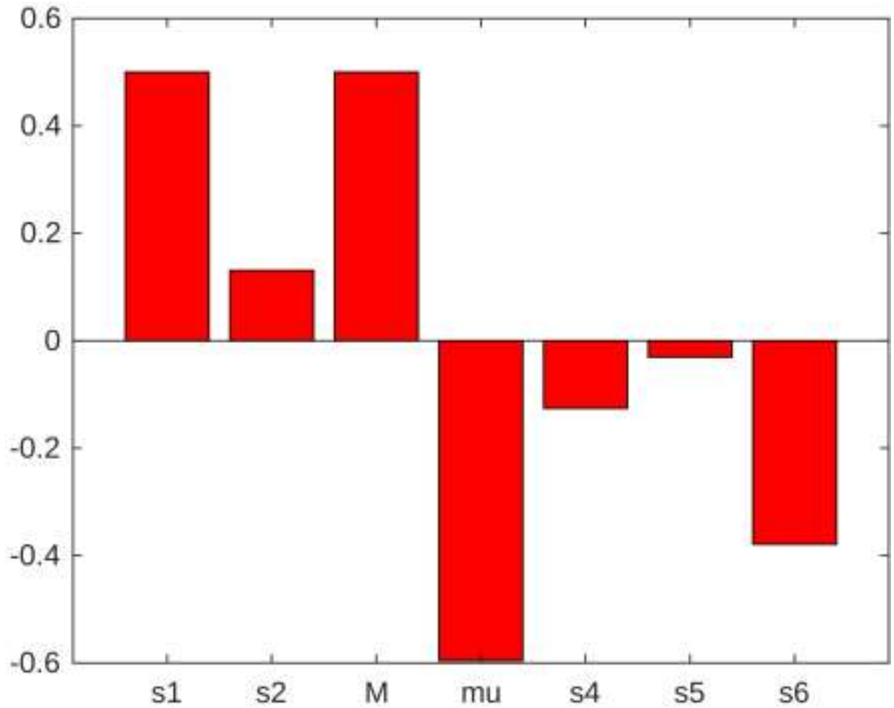

Fig. 4. Visual illustration of the sensitivity analysis of the key parameters

5. Results and Discussion

The outcomes of the proposed methodologies are illustrated in the subsequent Figures. $C_{nu} = 1000$, individuals $C_{eu} = 18$ Individulas, $C_{ru} = 18$ individuals, $C_{au} = 18$ Individuals, and $C_{hu} = 10$ individuals, and $C_{pu} = 10$ individuals are the starting numbers considered. We employ the fourth-order Runge-Kutta method (RK4) with MATLAB to produce the visual findings of the suggested model, as RK4 is a widely utilised computational technique for solving "ordinary differential equations (ODEs)". RK4 is a commonly often employed numerical approach for estimating values at all iterations within the algorithms.

The 10 first-order ordinary differential equations with boundary circumstances are resolved employing three distinct strategies. Comparative data in [53] indicate that the RK4 method performs better than in all scenarios and disciplines. In the intervening period, RK4 necessitates less computational time to evaluate the reduction in the worldwide imperfection in the computational solution, whereas [54] indicates that RK5 and RK8 have been less effective than RK4.

Figure 5 indicates that roughly 1000 persons are expected to be under control within just 25 from the experimental cohort of the updated (NERAPH) model. Conversely, Figure 6 demonstrates that the experimental cohort of the preceding (NERAH) model may attain 917 recovered persons within 47 days.

Similarly, Figure 7 demonstrates that the recreational category belonging to the NERAPH model could be controlled during 80 days, leading to the rehabilitation of 647 persons. Conversely, Figure 8 illustrates that the recreational category under the NERAH model can facilitate the rehabilitation of 596 persons over 88 days. Figure 9 illustrates the overall recovery of 206 persons over 105 days from the addicted category of the NERAPH model, whereas the addicted class of the NERAH approach obtains roughly 94 individuals within 99 days, as seen in Figure 10. Figure 11 illustrates the overall recovery of 18 persons over 113 days from the hospitalised category of the NERAPH model, but the hospitalised class of the NERAH approach may achieve about 18 individuals within 137 days, as seen in Figure 12. Figure 13 presents the findings of the prisoners class from the modified NERAPH model, indicating the recovery of 119 persons over a span of 120 days.

Moreover, a comparison of the cumulative results of the previous (NERAH) model with the revised (NERAPH) model reveals that the prior model attained an average recovery of 27.38 individuals per day, whereas the revised model exhibits a substantially better recovery rate of 51.19 individuals per day. Thus, the NERAPH model demonstrates faster convergence and best recovery performance relative to the NERAH model.

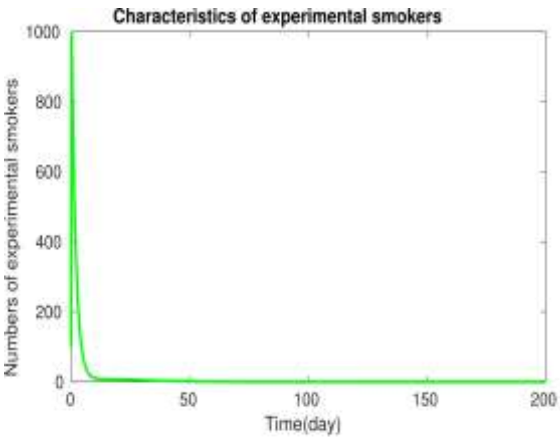

Fig. 5. Visual results of the experimental smokers from the updated (NERAPH) model

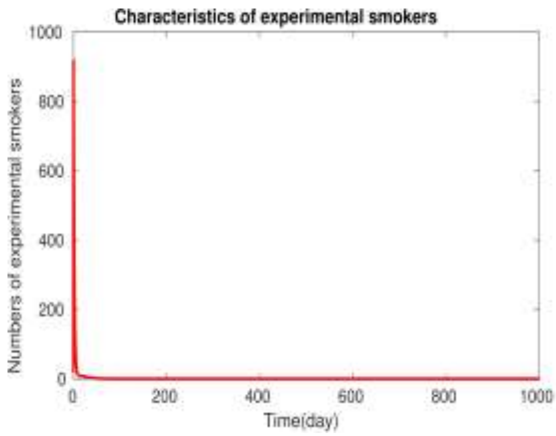

Fig. 6. Visual results of the experimental smokers from the prior (NERAH) model

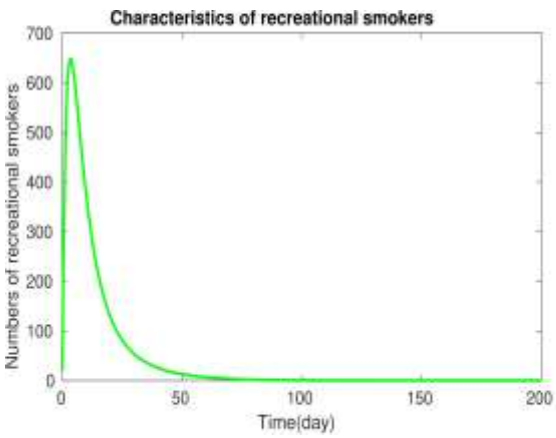

Fig. 7. Visual results of the recreational smokers from the updated (NERAPH) model

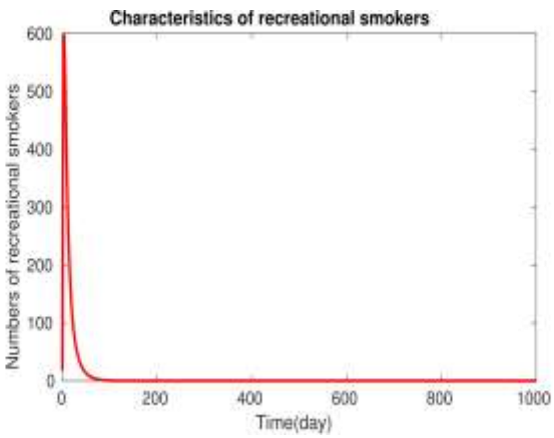

Fig. 8. Visual results of the recreational smokers from the prior (NERAH) model

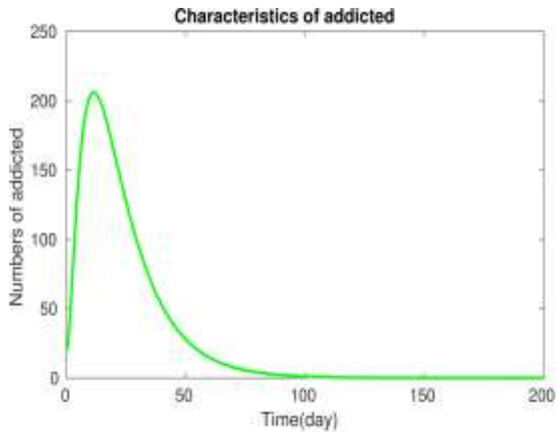

Fig. 9. Visual results of the addict's from the updated (NERAPH) model

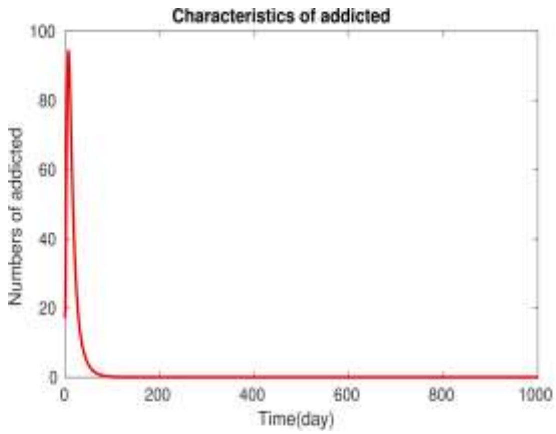

Fig. 10. Visual results of the addict's from the prior (NERAH) model

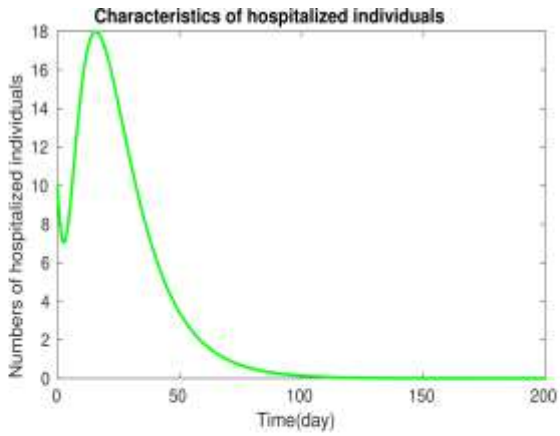

Fig. 11. Visual results of the hospitalized individuals from the updated (NERAPH) model

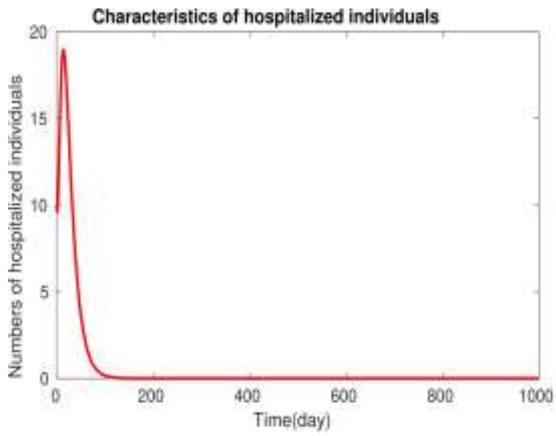

Fig. 12. Visual results of the hospitalized individuals from the prior (NERAH) model

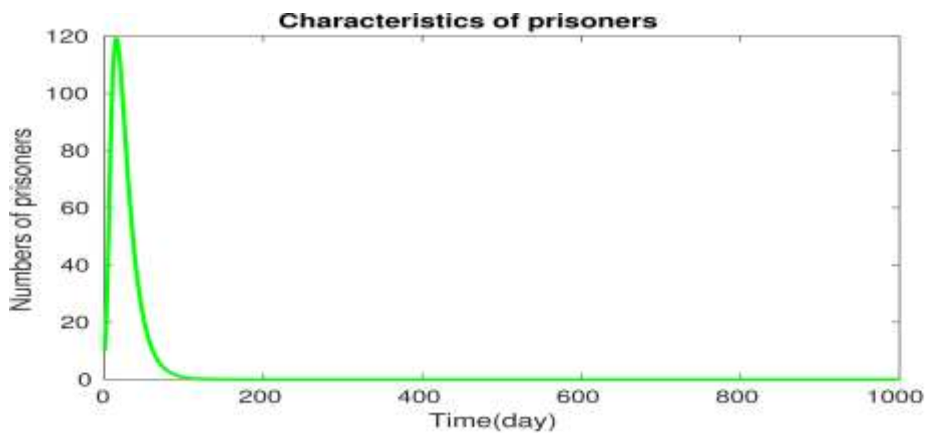

Fig. 13. Visual results of the additional prisoner's class from the updated (NERAPH) model

6. Conclusion

The current research's conclusive statements provide an improved mathematical approach to differentiating across cannabis users and non-users. The focus was on reducing addiction, combined with the establishment of a new category called the "prisoner's class." The study conducted a thorough comparison of results from these classes inside both the existing (NERAH) model and the improved (NERAPH) model for validation purposes. The comparison research provided insights into the effectiveness of each model in regulating cannabis use among the general population. The analysis highlighted the fundamental boundaries on the population, visible from the beginning phase (the invariant component), when the total number of people was discovered to be positively consistent. The essential role of the preliminary cannabis transmission rate ($\mathcal{R}_0$) was highlighted, accompanied by a comprehensive biological investigation clarifying its significance. The proposed model was subjected to numerical simulation using the fourth-order Runge-Kutta technique. This research resulting enhancing the effectiveness of the NERAH model by integrating "first-order non-linear ordinary differential equations" in the form a new compartment known as prisoners class. The suggested updated NERAPH framework was successfully verified and validated for improved performance, as demonstrated in its visual representation and mathematical segments, supported by sensitivity analysis. In conclusion, based on the results obtained from the thorough validation methods, we confirm the validity of the revised (NERAPH) model and its performance considered more effective as compared to the existing (NERAH) model. This model demonstrates reliability as a tool for future initiatives to decrease cannabis consumption in the general population. Future study efforts may utilise existing approaches such as the 'Optimal Control Problem' and 'Threshold Conditions,' which are crucial for minimising the length of time and costs of properly managing cannabis use.

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