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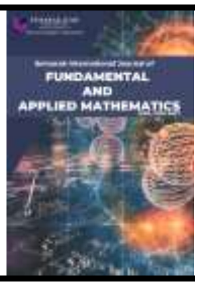

Numerical Analysis for the Fluid Flow and Heat Transfer of Free Convection Flow in a Porous Medium under Viscoelastic Properties

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ABSTRACT

The demand for research on boundary layer fluid flow issues in the fluid dynamic has led to the enhancement of fluid characteristics and heat transfer. One of the possible solutions to improve the heat transfer rate is through the integration of the existing model into a new model by incorporating a new concept or component of the fluid. Therefore, this paper presents numerical analysis of the free convection boundary layer flow of Brinkman-viscoelastic model through a horizontal circular cylinder (HCC) saturated in porous medium. The flow is considered flowing over a HCC under consideration of the convective boundary condition (CBC). The governing partial differential equations (PDEs) were transformed to the simplest form of non-linear PDEs by employing suitable non-dimensional variables and non-similarity transformation. Subsequently, the PDEs were solved by utilizing the finite difference method, named Keller-box method (KBM) and the coding was performed using *MATLAB* software. In a limited case, the current numerical results and earlier reports were compared to validate the problem model. According to the results, the velocity decreased and the temperature rose as the viscoelastic and Brinkman parameters were raised. Furthermore, increasing the viscoelastic and Brinkman parameters reduced skin friction and Nusselt number. The viscoelastic parameter was the only one that could maintain boundary layer separation. These theoretical results and the parameters in this investigation gave a significant effect in heat transfer process which can be used to assist engineers in making decisions and conducting experimental investigations.

1. Introduction

The exploration of fluid flow and heat transfer in porous materials with viscoelastic characteristics

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is critical for various engineering and environmental contexts. Natural phenomena, such as air conditioning systems, thermal storage systems, groundwater movement, and oil retrieval, are significantly influenced by the free convection flow occurring in such medium, which is propelled by buoyancy forces caused by temperature disparities. A thorough comprehension of these processes necessitates meticulous numerical analysis, with computational methodologies assuming a pivotal role. Utilizing mathematical models and numerical techniques allows researchers to delve into the intricate interactions between fluid dynamics, heat exchange, and viscoelastic traits within porous structures. This article endeavors to offer an extensive investigation into the complexities of free convection flow in porous media endowed with viscoelastic attributes. Through the implementation of advanced numerical algorithms and thorough analysis, the fundamental mechanisms dictating fluid movement and thermal conveyance in these environments are elucidated, thereby contributing to both the advancement of scientific knowledge and the optimization of industrial processes.

It is evident that the topic of free convection flow has attracted significant attention in a variety of industrial and technical applications. Molla *et al.,* [1] conducted an investigation into the free convection flow from an isothermal HCC in a viscous fluid and discovered that the viscosity of a fluid is directly proportional to its temperature. Molla *et al.,* [2] subsequently expanded the study by incorporating the internal heat generation effect. According to both investigations, the boundary layer did not separate as it passed through the cylinders. In addition, Molla *et al.,* [3] continued their investigation into the impact of radiation on fluids, which revealed that the heat transfer rate and the thickness of the thermal boundary layer were both enhanced by the effects of radiation. The boundary layer equations in the preceding studies were resolved through the application of the finite difference method. Additionally, Zainuddin *et al.,* [4] examined the impacts of free convection flow over a heated HCC in a stationary fluid concerning both radiation and heat generation. In contrast to the radiation effect, it was demonstrated that heat generation substantially influenced fluid temperature. Subsequently, Novomestský *et al.,* [5] conducted experiments in an environmental chamber to examine the heat transfer distribution of a heated horizontal cylinder adjacent to the wall, discovering that the proximity of the horizontal cylinders to the side wall may influence heat transfer. Swalmeh *et al.,* [6] examined free convection over a heated HCC submerged in a viscous micropolar nanofluid composed of water and kerosene oil. The heat transfer coefficient of kerosene oil was more significantly influenced by the micro-rotation parameter value than that of water nanofluid. Yasin *et al.,* [7] investigated the influence of the magnetic field on free convection in ferrofluid at a lower stagnation point by analysing the flow and heat transfer characteristics of ferrofluid under varying magnetic parameters and ferroparticle volume fractions. They found that an increase in the strength of the magnetic parameter correlates with a decrease in the Nusselt number of the ferrofluid. Furthermore, Zokri *et al.,* [8] examined the free convection flow of Jeffrey nanofluid incorporating viscous dissipation effects, demonstrating that an increase in viscous dissipation mitigated boundary layer separation. The mathematical model of a hybrid nanofluid for free convection flow over HCC was examined by Mohamed *et al.,* [9], who demonstrated that an increased concentration of nanoparticles improved the skin friction coefficient. Moreover, high-density nanoparticles in the nanofluid usually cause a great degree of friction between the fluid and the cylinder surface, which damages the component surface.

The governing equations are mathematically formulated to represents the physical principles of fluid flow and heat transfer in fluid dynamics. In this context, earlier researchers have utilised various numerical techniques to address the boundary layer problem, including the Runge-Kutta Fehlberg Method, Homotopy Perturbation Method, Finite Difference Technique and Finite Element Method, respectively [10-14]. Specifically, two implicit finite difference methods that have been extensively employed to address ordinary differential equations (ODEs) and partial differential equations (PDEs) are the Crank-Nicolson method and KBM [15]. According to multiple scholars [16,17], the KBM is a highly effective method due to its adaptability to new problem classes, rapid net variation, ease of achieving higher-order accuracy, productivity, and suitability for solving non-linear PDEs. The KBM can be modified to address an issue in any sequence. Numerous boundary layer problems have been addressed utilising the KBM in various published works. This encompasses Nazar *et al.,* [18], Tham *et al.,* [19], Mahat *et al.,* [20, 21], Mohamed *et al.,* [22], Ganesh and Sridha [23], and Bhat *et al.,* [24].

Currently, multiple studies are required to analyse the convection flow and heat transfer mechanisms in the context of a porous medium, incorporating viscosity and elasticity characteristics. The Brinkman model was established as a classic porous medium model utilised for incompressible fluid flow in highly porous materials. Therefore, it is beneficial to examine the boundary layer fluid dynamics and key factors that can enhance heat transfer and postpone the separation process. Additionally, the literature has yet to investigate the flow of Brinkman-viscoelastic fluid. Consequently, this study will address the free convective flow on the HCC of Brinkman-viscoelastic fluid embedded in a porous region, thereby addressing the gap.

2. Research Diagram

The research flow begins with the problem formulation followed by numerical method, which is programmed in MATLAB software, validation of results by comparing with the previous established research, and finally the analysis of results and discussion. The summaries of research methodology have been illustrated in Figure 1.

3. Problem Formulation

Consider a steady free convection boundary layer flow past the HCC of radius a embedded in a porous medium, as illustrated in Figure 2. The $\bar{x}$ axis and $\bar{y}$ axis are respectively measured in parallel with the cylinder surface and perpendicular to it under the influence of CBC. In addition, T_∞ denotes the ambient temperature and g denotes the gravitational acceleration.

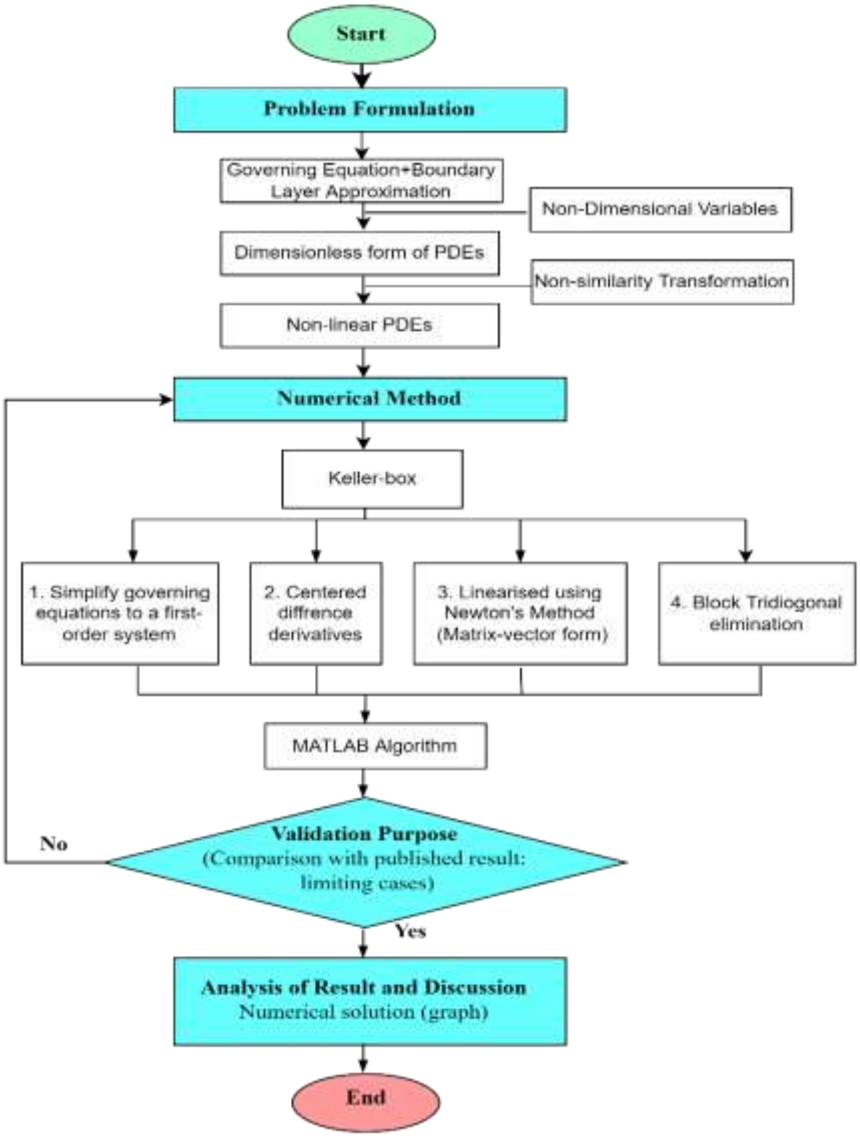

Fig. 1. Research diagram

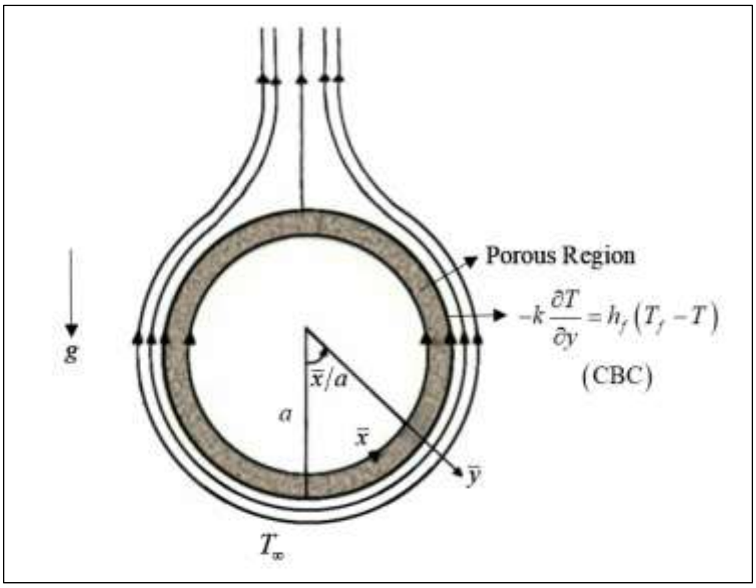

Fig. 2. Geometry model

For this current problem, the governing boundary layer equations for Brinkman-viscoelastic fluid were obtained using boundary layer approximation by dropping some insignificant terms in the equations. Therefore, the governing equations are as follows

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\frac{\mu}{K}\,\bar{u} = -\frac{dp}{d\bar{x}} + \frac{\mu}{\phi}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + k_0\left[\bar{u}\,\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2} + \bar{v}\,\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right] - \rho g \sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\,\frac{\partial T}{\partial \bar{x}} + \bar{v}\,\frac{\partial T}{\partial \bar{y}} = \alpha_m \frac{\partial^2 T}{\partial \bar{y}^2}. \tag{3}$$

with the appropriate convective boundary conditions as below

$$\begin{aligned} &\bar{v} = 0, \quad \bar{u} = 0, \quad -k\frac{\partial T}{\partial \bar{y}} = h_f\left(T_f - T\right) \quad at \;\; \bar{y} = 0, \\ &\bar{u} \to 0, \quad \frac{\partial \bar{u}}{\partial \bar{y}} \to 0, \quad T \to T_\infty \qquad\qquad as \;\; \bar{y} \to \infty. \end{aligned} \tag{4}$$

Next, the non-dimensional variables from Chamkha *et al.,* [25] as below

$$x = \frac{\bar{x}}{a}, \quad y = Ra^{1/2}\left(\frac{\bar{y}}{a}\right), \quad u = \frac{\bar{u}}{U_c}, \quad v = Ra^{1/2}\,\frac{\bar{v}}{U_c}, \quad \theta = \frac{(T - T_\infty)}{(T_f - T_\infty)}. \tag{5}$$

are introduced to convert the system of Eq. (1) – (4) into Eq. (6) – (9)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$\frac{\partial u}{\partial y} = \Gamma\frac{\partial^3 u}{\partial y^3} + k_1\left[\begin{aligned} &u\frac{\partial^4 u}{\partial x\partial y^3} + \frac{\partial^3 u}{\partial x\partial y^2}\frac{\partial u}{\partial y} + v\frac{\partial^4 u}{\partial y^4} \\ &+\frac{\partial^3 u}{\partial y^3}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial^3 u}{\partial x\partial y^2} - \frac{\partial^2 u}{\partial x\partial y}\frac{\partial^2 u}{\partial y^2} \\ &+\frac{\partial u}{\partial x}\frac{\partial^3 u}{\partial y^3} + \frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial x\partial y} \end{aligned}\right] + \frac{\partial \theta}{\partial y}\,\sin x, \tag{7}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2}, \tag{8}$$

together with the transformed boundary condition

$$\begin{aligned} &u = 0, \quad v = 0, \quad \theta' = -Bi\left(1-\theta\right) \quad \text{at} \;\; y = 0, \\ &u \to 0, \quad \frac{\partial u}{\partial y} \to 0, \quad \theta \to 0 \quad \text{as} \;\; y \to \infty. \end{aligned} \tag{9}$$

Moreover, the physical parameters involved in this study are declared as follows: $\Gamma = \frac{Da}{\phi} Ra$, $Da = \frac{K}{a^2}$, $Ra = \frac{\rho_\infty g k \beta (T_w - T_\infty) a}{\alpha_m \mu}$, $U_c = \frac{\rho_\infty g k \beta (T_w - T_\infty)}{\mu}$, $k_1 = \frac{k_0 K U_\infty Ra}{\mu a^3}$ and $Bi = \frac{h_f}{k}\sqrt{Ra}$ denotes the Brinkman parameter, Darcy number, Rayleigh number, characteristic velocity, viscoelastic parameter and Biot number, respectively.

Subsequently, the non-similarity transformation introduced by Tham *et al.,* [26] and Kanafiah *et al.,* [27] as below

$$\psi = x\, f(x,y),\ \theta = \theta(x,y),\ u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}, \tag{10}$$

were used for Eq. (6)-(9) in which ψ and θ are defined as the stream function and fluid temperature, respectively. Next, u and v in Eq. (10) can be derived as

$$u = x\frac{\partial f}{\partial y},\quad v = -x\frac{\partial f}{\partial x} - f. \tag{11}$$

Note that Eq. (6) has been completely satisfied. Hence, the following PDEs are attained

$$\begin{aligned} & f' - \Gamma f''' - k_1\left[2ff''' - ff^{(iv)} - (f'')^2\right] - \theta\frac{\sin x}{x} = \\ & xk_1\left[f'\frac{\partial f'''}{\partial x} - \frac{\partial f}{\partial x} f^{(iv)} - f''\frac{\partial f''}{\partial x} + \frac{\partial f'}{\partial x} f'''\right], \end{aligned} \tag{12}$$

$$\theta'' + f\,\theta' = x\left(f'\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\theta'\right) \tag{13}$$

with respect to the following boundary conditions

$$\begin{aligned} & f(0) = 0,\quad f'(0) = 0,\quad \theta'(0) = -Bi\left(1 - \theta(0)\right), \\ & f'(\infty) \to 0,\quad f''(\infty) \to 0,\quad \theta(\infty) \to 0. \end{aligned} \tag{14}$$

In reference to Tham *et al.,* [26] and Mahat *et al.,* [21], the skin friction coefficient and Nusselt number after employing some derivation processes are given as below:

$$C_f Ra^{1/2} / \Pr = xf''(0),\quad Nu_x Ra^{-1/2} = -Bi\left(1 - \theta(0)\right). \tag{15}$$

Finally, the KBM was used to obtain the numerical solutions of Eq. (12) to Eq. (14). This numerical approach has been utilized by many scholars to solve the convective boundary layer issues as mentioned in literature. The primary sources for this technique are presented in Jamshed *et al.,* [28] and Ganesh & Sridha [23], which include the following four main steps:

Step 1: Convert the PDEs to first order system

The new dependent variables for velocity and temperature profiles are initiated as $f' = u(x,y),\ f'' = v(x,y),\ f''' = p(x,y), \theta = s(x,y)$ and $\theta' = t(x,y)$ to convert the non-linear PDEs of Eq. (12) and Eq. (13) into first order system.

Step 2: Discretize the equation and write in finite difference form

Firstly, the system from step 1 are descretised in n and j terms where n and j are the numerical sequence indicating the coordinate position. Then, the first order system is written in finite difference form by employing the central difference formula about the midpoint.

Step 3: Newton's method

The finite difference forms are linearized by utilising the Newton's method together with some iterations. Next, by performing some algebraic alteration, the linear equations are generated in a block matrix form.

Step 4: Block tri-diagonal elimination technique

Finally, the system in a block matrix form is solved using the block tri-diagonal elimination method to compute the significant parameter in this study.

4. Validation of Results

The numerical results of Eqs. (12) to (14) were obtained using the KBM. MATLAB software was employed to execute the numerical computations and algorithms. Specifically, computations were carried out by selecting the finite boundary layer thickness of $y = 20$ and step size, $\Delta y = 0.02$ and $\Delta x = 0.01$. The Brinkman and viscoelastic parameters were calculated to ascertain the fluid flow's velocity and temperature, as well as the local skin friction coefficient and Nusselt number.

The present results were compared with the numerical outcomes of Nazar *et al.,* [18] and the precise solution of Harris *et al.,* [29] in the absence of k_1 at the stagnation point $(x \cong 0)$, incorporating the added constant, A and coefficient, B in Eq. (12). Table 1 presents the current model at the stagnation point, compared with the existing equation under several restricted scenarios. The comparative values for $f''(0)$ were validated (refer to Table 2). The numerical values in Table 2 indicate that the current numerical results align closely with previous values; therefore, the numerical algorithm proposed in this study is deemed accurate. The parameter values were chosen based on the possibility of the velocity and temperature profiles satisfying the boundary conditions. Therefore, in the entire work, the values of the viscoelastic and Brinkman parameters were between $1 \le k_1 \le 4$ and $0.1 \le \Gamma \le 0.7$, respectively.

Table 1
Comparison of model at stagnation point

Author	Model (momentum)	Limiting cases
Current	$f' - \Gamma f''' - k_1\left[2 f f''' - 2 f f^{(iv)} - (f'')^2\right] - A - B\theta = 0$	$k_1 = 0, A = 1, B = 0$
Nazar *et al.,* [18]	$f' - \Gamma f''' - 1 - \lambda\theta = 0$	$\lambda = 0$
Harris *et al.,* [29]	$f' - \Gamma f''' - 1 - \lambda\theta = 0$ Exact equation: $f''(0) = \frac{1}{\sqrt{\Gamma}}$	$\lambda = 0$

Table 2
Comparison of $f''(0)$ for various Γ

Γ	$\lambda=0$ $\left(with\ k_1=0, A=1, B=0\right)$		
	Harris *et al.,* [29]	Nazar *et al.,* [18]	Current
	$f''(0)$	$f''(0)$	$f''(0)$
0	-	-	-
0.1	3.1622	3.1623	3.1622
0.2	2.2360	2.2361	2.2360
0.3	1.8257	1.8257	1.8257

5. Analysis of Results and Discussion

This study examines the velocity and temperature profiles at the stagnation point. Consequently, at the minimum stagnation point of the cylinder, the values in Eq. (12) and Eq. (13) approach zero, thereby transforming the equations into ordinary differential equations (ODEs). Figures 3–10 show the velocity and temperature distributions, skin friction coefficient, and Nusselt number for different values of k_1 and Γ. Figure 3 shows the velocity profiles for different values of k_1. The figure demonstrates that the impact of k_1 resulted in a velocity of $y=0$ to 2.94, subsequently followed by a reversal in the velocity profiles, which increased in the direction of the free stream. This circumstance is associated with dual physical properties known as viscosity and elasticity, which aid in resisting fluid motion. Meanwhile, the temperature profile increased as the parameter, k_1 increased in Figure 4 due to the convection process when the temperature gradually drops to the ambient temperature.

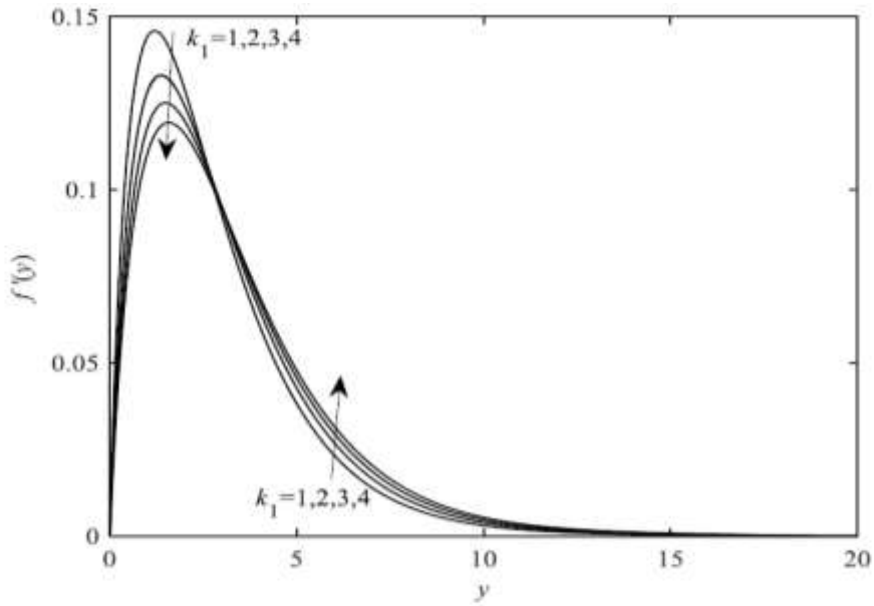

Fig. 3. Velocity profile, $f'(y)$ for a variety of k_1 with $\Gamma=0.1$ and $Bi=0.1$

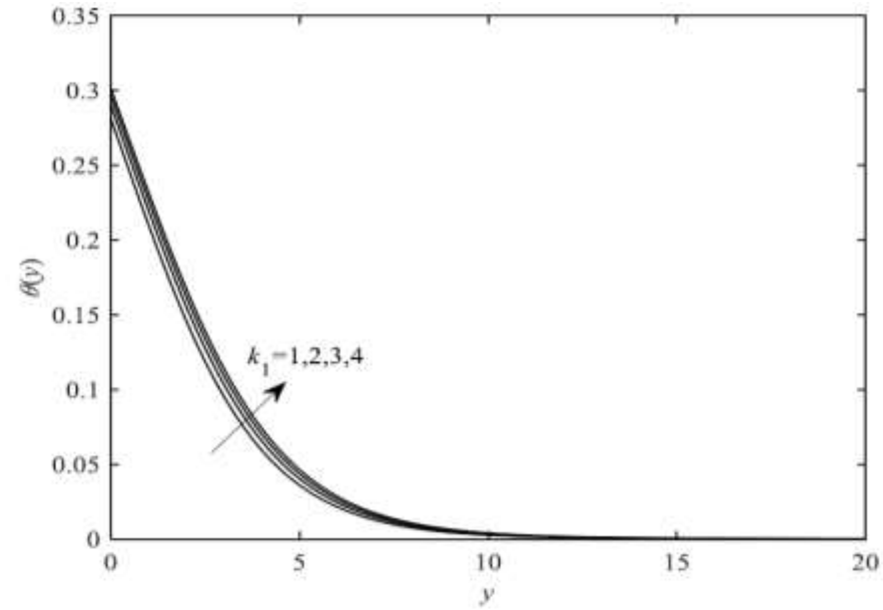

Fig. 4. Temperature profile, $\theta(y)$ for a variety of k_1 with $\Gamma = 0.1$ and $Bi = 0.1$

A comparable velocity profile behaviour to that depicted in Figure 3 was observed in Figure 5 when the value of increased. A decline in fluid velocity was noted; subsequently, the graph inverted, resulting in an increase in fluid flow as the variable rose from 0.1 to 0.7. This outcome is attributable to the influence of the drag force, which caused a marginal elevation in temperature profiles, as seen in Figure 6.

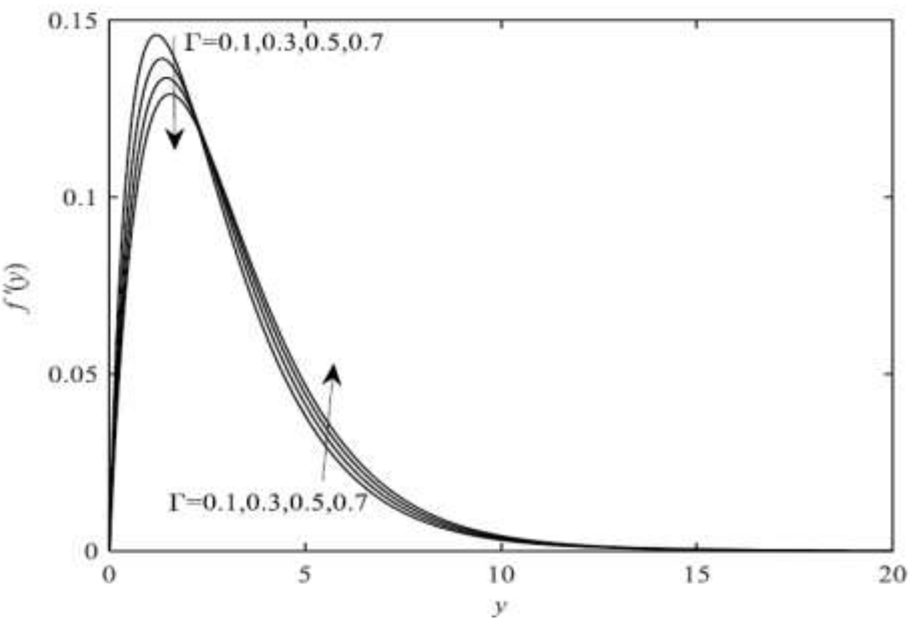

Fig. 5. Velocity profile, $f'(y)$ for a variety of Γ with $k_1 = 1$ and $Bi = 0.1$

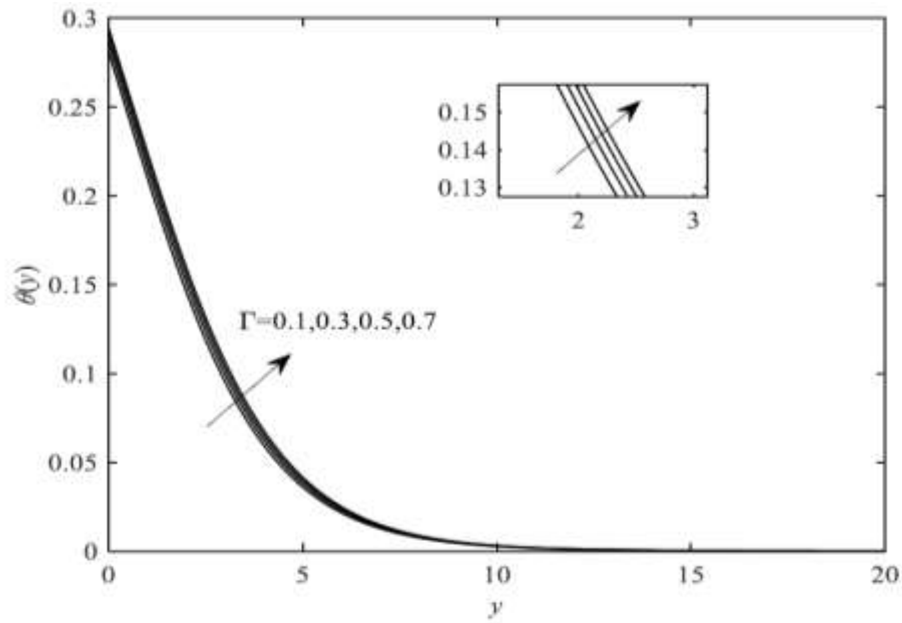

Fig. 6. Temperature profile, $\theta(y)$ for a variety of Γ with $k_1 = 1$ and $Bi = 0.1$

Next, based on Figure 7, the increment value of k_1 from 1 to 4 decreased $C_f Ra^{1/2} / \mathrm{Pr}$ significantly, similar to $Nu_x Ra^{-1/2}$, as shown in Figure 8. This observation aligns with previous findings that the increase in velocity and temperature profiles is solely attributable to the enhancement of the viscoelastic parameter, thereby diminishing friction and heat transfer rates at the surface. However, for the specific value of k_1, the performance of $C_f Ra^{1/2} / \mathrm{Pr}$ increased while $Nu_x Ra^{-1/2}$ experienced reduction behaviour. For instance, at $k_1 = 1$, the $C_f Ra^{1/2} / \mathrm{Pr}$ increased (see Figure 7), whereas $Nu_x Ra^{-1/2}$ decreased as shown in Figure 8. In addition, these figures show that as the value of k_1 increased, it took quite longer for the boundary layer to separate from the cylinder surface. Here, it can be concluded that the viscoelastic parameter possesses a strong impact of pressure drags on the cylinder, which can serve as useful information in engineering applications.

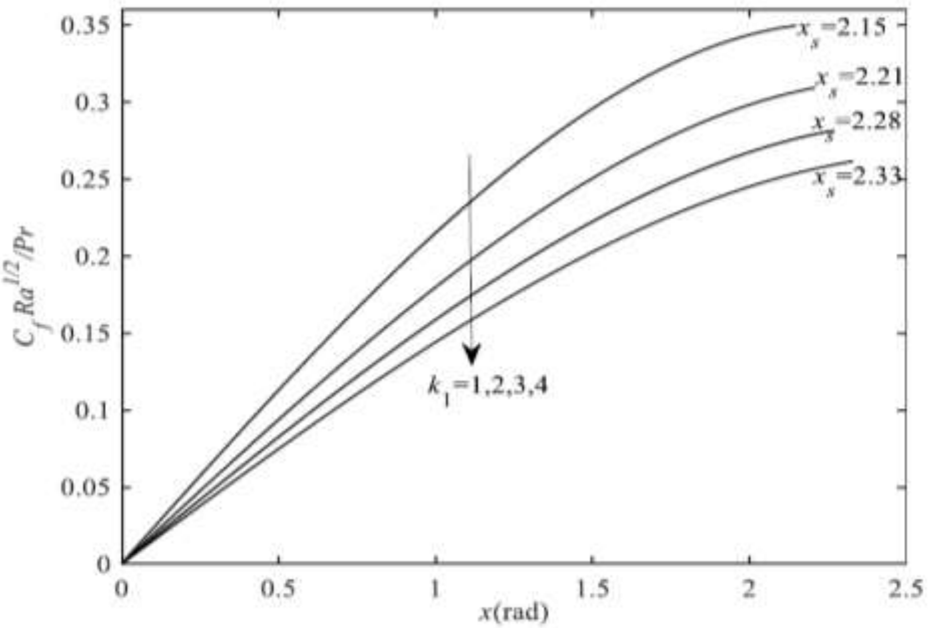

Fig. 7. Skin friction coefficient, $C_f Ra^{1/2} / \mathrm{Pr}$ for a variety of k_1 with $\Gamma = 0.1$ and $Bi = 0.05$

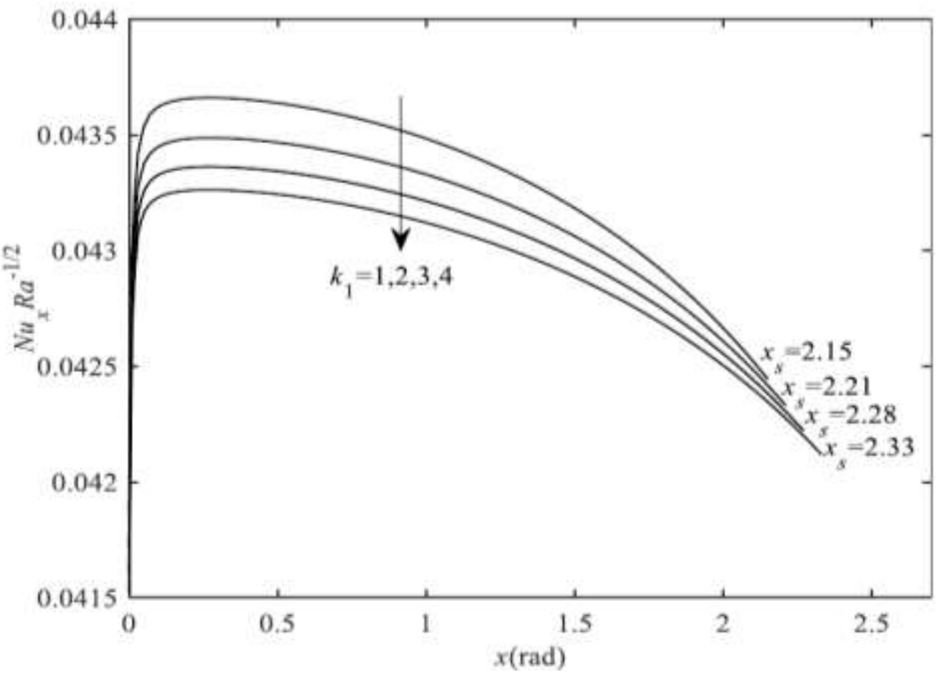

Fig. 8. Nusselt number, $Nu_x Ra^{-1/2}$ for a variety of k_1 with $\Gamma = 0.1$ and $Bi = 0.05$

The graphs for the skin friction coefficient and Nusselt number are plotted in Figure 9 and Figure 10 for the different values of Brinkman parameter, Γ. Similar behaviour to the ones in Figure 7 and Figure 8 was noticed such that $C_f Ra^{1/2} / \mathrm{Pr}$ and $Nu_x Ra^{-1/2}$ deteriorated as Γ increased from 0.1 to 0.7. Nevertheless, the boundary layer separation behaves reversely by incrementing the value of Γ. For example, in Figure 9 and Figure 10, an increase in Γ expedited the separation of the boundary layer flow from the cylinder. This phenomenon causes less pressure to drag, which is very important in manufacturing processes to produce the desired output.

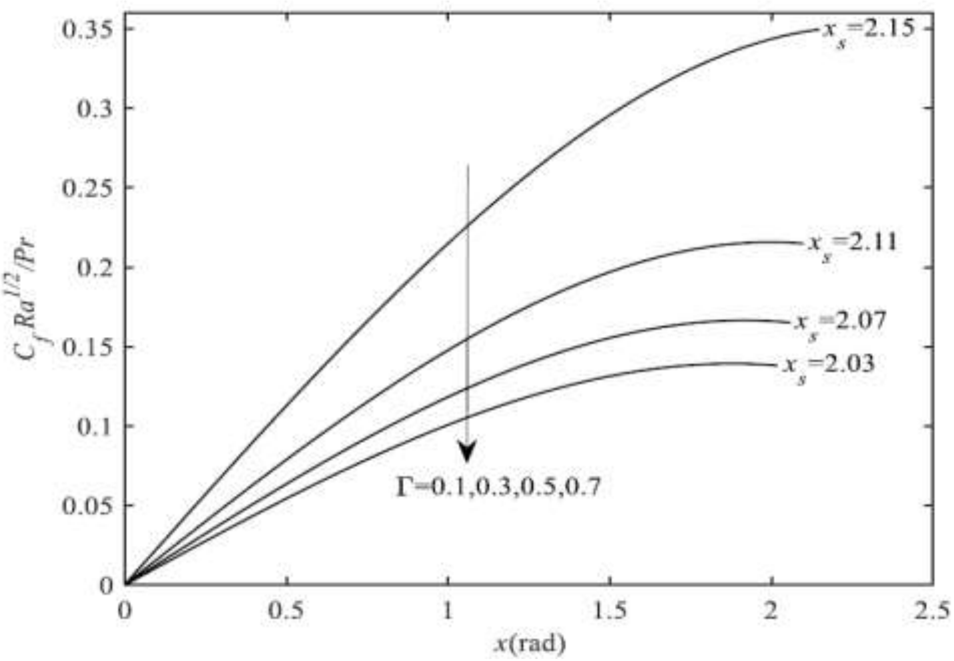

Fig. 9. Skin friction coefficient, $C_f Ra^{1/2} / \mathrm{Pr}$ for a variety of Γ with $k_1 = 1$ and $Bi = 0.05$

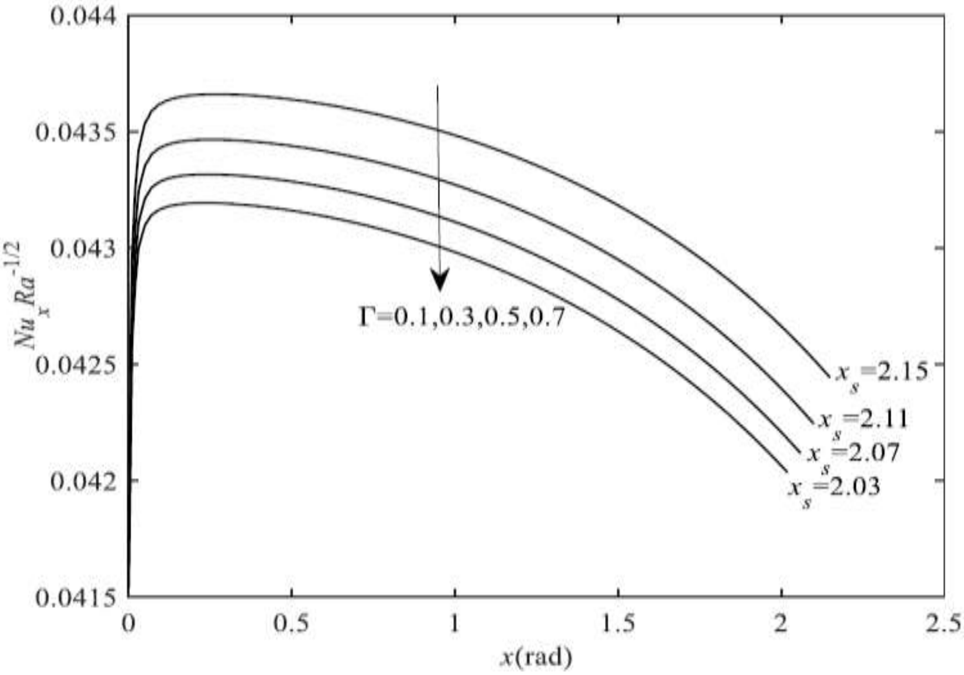

Fig. 10. Nusselt number, $Nu_x Ra^{-1/2}$ for a variety of Γ with $k_1 = 1$ and $Bi = 0.05$

5. Conclusions

Overall, this study has numerically investigated the free convection of Brinkman-viscoelastic fluid passing over a HCC placed in porous medium for CBC cases. Particularly, this research has analysed the impact of Brinkman and viscoelastic parameters towards the velocity, temperature, skin friction coefficient as well as Nusselt number.

Significantly, the viscoelastic and Brinkman parameters demonstrated contradict behaviour for the velocity and temperature profiles. Evidently, increasing both parameters would reduce the velocity of the fluid while increasing the temperature. Furthermore, the skin friction coefficient and Nusselt number reduced with the increase in the viscoelastic and Brinkman parameter. Moreover, the parameter viscoelastic in this investigation plays an important role in slowing down the boundary layer separation of the flow which can be useful information in many engineering applications.

A comprehensive of numerical results can be used as a reference for future research or for guiding experimental process. Due to a lack of previous research, the study focuses solely on convective boundary layer flow over a bluff body saturated in a porous medium, and this is limited to HCC without considering any physical effects. Thus, in the future, it would be interesting to consider other geometry such as sphere and cone, and also can consider the physical effects such as magnetic field, thermal radiation and chemical reactions.

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