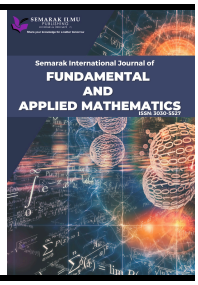
Semarak International Journal of
Fundamental and Applied Mathematics

Journal homepage:
https://semarakilmu.my/index.php/sijfam/index
ISSN: 3030-5527



VCD_SPT : A MATLAB Application for Determining Volume, Conductivity and Depolarization Factor of a Conducting Spheroid from Its Polarization Tensor

Taufiq Khairi Ahmad Khairuddin[1,*], Nurul Tasnim Abdul Latiff [1], Nurhazirah Mohamad Yunos[2], Nor Muhainiah Mohd Ali[1], Mukhiddin Muminov[3]

[1] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[2] Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Cawangan Johor, Kampus Pasir Gudang, 81750 Masai, Johor, Malaysia
[3] Mathematical Faculty, Samarkand State University, Samarkand, 140104, Uzbekistan

ARTICLE INFO

Article history:
Received 20 August 2026
Received in revised form 27 September 2026
Accepted 30 September 2026
Available online 12 October 2026

Keywords:
Electric field; matrices; system of equations; Cramer's Rule; Newton's Method

ABSTRACT

In electric field-based imaging, determining a spheroid from the first order PT associated with the perturbation caused by a hidden conducting object can provide a useful representation of the object. The volume, conductivity, and depolarization factor of the resulting spheroid can then be used to describe the size, electrical properties, and shape of the conducting object. However, determining the parameters of a conducting spheroid from a given first order PT involves mathematical computations that can be complex and time-consuming when performed manually. This study aims to develop a MATLAB application for determining the conductivity and volume of a conducting spheroid from its first order PT, providing an efficient and interactive tool to simplify the computations and facilitate the characterization of the spheroid. The application called as VCD_SPT is developed using MATLAB App Designer to provide an interactive platform that integrates analytical formulations with numerical computation, while supporting interface design, callback programming, and input validation. After that, the developed application is validated using selected cases from previous studies to assess its accuracy and reliability. With the developed application, the characterization of conducting spheroids from their first order PT can be performed using a practical and user-friendly tool, with potential applications and commercialization opportunities in research, education, and industry.

1. Introduction

Polarization tensor (PT) arises in applications such as electric-field based imaging [1-6] and metal detection [7-10]. In electrical impedance tomography (EIT), PT can be used to help locate tumours based on their abnormal electrical conductivity, while in landmine detection, electromagnetic responses can be analyzed using PT to distinguish metallic objects from the surrounding soil. The core

* *Corresponding author.*
E-mail address: taufiq@utm.my

problem in these applications is to interpret how an object perturbs an applied electric or electromagnetic field in order to deduce its intrinsic properties, such as its shape, size, and material composition. The PT provides a powerful mathematical framework for addressing this problem, although its formulation for electric fields differs from that for electromagnetic fields. For an object of a given shape, its first order PT acts as a compact descriptor, represented by a matrix [1,11-12], whose entries encode information about its geometry, conductivity contrast with the surrounding medium, and polarization response to the applied field. In this case, computing and characterizing the first order PT associated with an object whose geometry and conductivity are known are useful for understanding its electrical response and developing methods for object detection, shape estimation, and material characterization [1,3,5,7,9-13]. On the other hand, finding an equivalent ellipsoid from a given first order PT is also useful for characterizing the geometry and electrical properties of an unknown or hidden object. Some properties investigated related to the scaling effects on the volume and semi axes of conducting spheroid based on its first order PT [14], the orientation of a rotated ellipsoid from its first order PT [15] and the determination as well as the relationship between the conductivity and depolarization factor of a conducting spheroid [16].

Determining the parameters of an equivalent ellipsoid from a given first order PT involves mathematical computations that can be complex and time-consuming when performed manually. Therefore, there is a need for an accessible computational tool that integrates the established mathematical formulation into a systematic procedure for determining the relevant spheroid parameters. Hence, the objective of this study is to develop a MATLAB application for determining the conductivity and volume of a conducting spheroid, an ellipsoid with two equal semi-axes, from its first order PT. The application simplifies the necessary computations by providing an efficient and interactive means of obtaining the spheroid parameters, making the characterization process more accessible and less prone to computational errors.

This study employs a computational approach to develop a MATLAB Graphical User Interface (GUI) for determining the volume, conductivity, and depolarization factor of a conducting spheroid from its first order polarization tensor, assuming that the spheroid perturbs an applied electric field. The GUI provides an interactive platform that integrates analytical formulations with numerical computation, enabling users to investigate two principal cases: (i) prescribed conductivity, where the volume and depolarization factor are determined, and (ii) prescribed volume, where the effective conductivity and depolarization factor are determined. The volume represents the size of the spheroid, while the conductivity describes its electrical material property and the depolarization factor [17] characterizes its shape and dimension. The resulting spheroid serves as an equivalent representation of an object that shares the same first order PT, allowing its dominant geometric and electrical characteristics to be described using a simple spheroidal model. The significance of the study lies in providing a practical computational implementation of the existing mathematical theory, thereby reducing the complexity of manual calculations and facilitating the characterization of an equivalent spheroidal object. The application is developed using MATLAB App Designer, which provides a structured environment for interface design, callback programming, numerical computation, and input validation.

The paper now proceeds as follows. The next section presents the main mathematical formulation used in the application. After that, Section 3 briefly describes the developed application. Then, several examples for validating the application are provided in Section 4. The final section concludes the paper by discussing the benefits and commercialization potential of the developed application.

2. Mathematical Formulation

Suppose that an applied electric field is perturbed by a conducting object whose conductivity has a constant contrast ratio k with respect to the surrounding background medium. The analysis of such field perturbations is relevant to practical applications such as medical imaging using EIT [1,17-19] and electrosensing by weakly electric fish [2-6]. In electrosensing fish, the resulting electrical response can be analyzed to characterize and recognize objects detected by the fish. Moreover, according to Ammari and Kang [1], the perturbation can be represented by an asymptotic series, whose leading terms are described by generalized polarization tensor (GPT). Here, the first order PT is the simplest and most fundamental form of the GPT and provides information about the object's geometry and conductivity contrast.

Let the first order PT, M associated with the conducting object be given by

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{bmatrix}, \tag{1}$$

where M_1 and M_2 are real-valued constants, such that either both M_1 and M_2 are positive or both are negative. In practice, the matrix can be obtained from field measurements (see [8] as an example) or through simulations using an explicit formula (see, for example, [11]). It has been shown in previous studies [5,16] that an equivalent spheroid can be determined from a given first order PT in the form of Eq. (1), and hence Eq. (1) is called as the spheroidal first order PT (SFOPT) (see Figure 1). Several studies have subsequently investigated methods for determining the semi-axes of the spheroid (and hence, volume and depolarization factors) from Eq. (1) under prescribed conductivity [20–22]. Similarly, previous study in Yunos *et al*., [16] presented a method for determining the conductivity and depolarization factors of the spheroid under prescribed volume. By setting Eq. (1) equal to the first order PT for spheroid given in Ammari *et al*., [1] (see [16] and [21]), the following system of equations relate the conductivity k, the volume |B| and also the depolarization factors, d_1 of the spheroid.

$$M_1(1-k)d_1 + (k-1)|B| = M_1, \tag{2}$$

$$\frac{M_2}{2}(k-1)d_1 + (k-1)|B| = M_2 + \frac{M_2}{2}(k-1). \tag{3}$$

In Eq. (2) and Eq. (3), the same symbols are retained from our previous publications to ensure consistency and facilitate comparison with the existing results later on.

Since M_1 and M_2 are given, the system of equations Eq. (2) and Eq. (3) contain three unknowns which are k, |B| and d_1. Thus, one of the unknowns can be prescribed based on prior knowledge or assumptions about the spheroid, allowing the other two unknowns to be determined. When k is prescribed, |B| and d_1 can be determined by solving the resulting system of linear equations. Meanwhile, when |B| is prescribed, k and d_1 can be determined by solving the resulting nonlinear system of equations. The method for prescribed conductivity was outlined, for example, in Khairuddin *et al*., [21], while Yunos *et al*., [16] recently presented the corresponding method for prescribed volume.

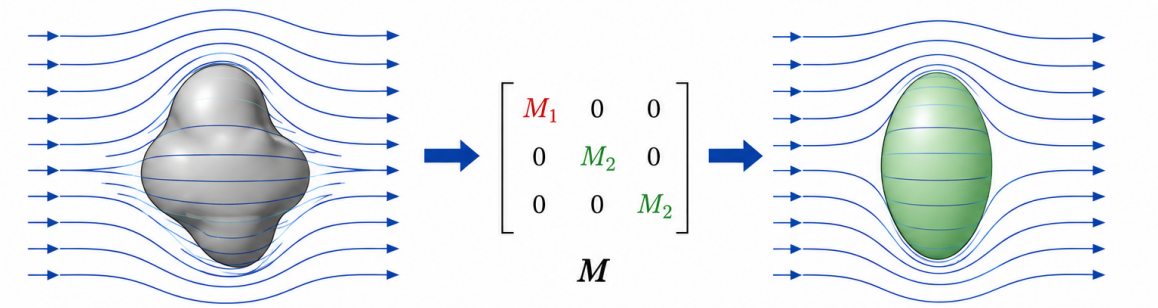


Fig. 1. Conceptual illustration of the perturbation of an applied electric field by a conducting object and its representation by an equivalent spheroid through the first order polarization tensor, M

These two formulations provide flexibility in characterizing an object when either its conductivity contrast or its volume is known or can be reasonably estimated. The resulting parameters provide information about the size, electrical properties, and shape of the spheroid represented by the given first order PT. This approach is therefore useful for obtaining a simplified spheroidal representation of an object from its electrical response, which is, the given first order PT as in Eq. (1).

In this study, a MATLAB application will be presented to facilitate the computation of both cases. In order to demonstrate the effectiveness of the proposed application, the mathematical formulations derived from Eq. (2) and Eq. (3) will not be discussed in detail. For detailed derivations and explanations of the formulations, interested readers may refer to Yunos *et al*., [16] and Khairuddin *et al*., [20].

3. The MATLAB Application

In our previous studies, most of our focus was on finding an ellipsoid or a spheroid from a given first order PT when the conductivity was fixed. In this case, not only were the volume and the depolarization factor obtained, but also the corresponding semi-axes of the ellipsoid or spheroid [20-22]. Only recently has our focus shifted to the case of fixed volume, specifically for spheroids [16]. Due to the possible relationship between conductivity, volume, and depolarization factor, our work then focuses on these three parameters without determining the semi-axes, especially since these three parameters seem sufficient to describe a spheroid. MATLAB applications for the fixed-conductivity and fixed-volume cases can be found in Nurhazirah *et al*., [23] and Abdul Latiff *et al*., [24]. Both applications were copyrighted by MyIPO under reference numbers CRLY2024J02487 and CRLY2025J08284, respectively. Now, the present study introduces a novel approach by combining both formulations into a single MATLAB application, providing a more comprehensive and flexible tool for characterizing a conducting spheroid from a given first order PT.

Figure 2 shows the flowchart used to develop the MATLAB application using MATLAB App Designer. Two menus are provided, which are, fixed conductivity and fixed volume. Under the fixed-conductivity option, the volume and depolarization factor of the spheroid are computed by solving the corresponding system of linear equations using Cramer's rule after the values of M_1, M_2 and k are specified. On the other hand, under the fixed-volume option, the conductivity and depolarization factor of the spheroid are computed by solving the corresponding nonlinear system of equations using Newton's method.

Next, Figure 3 shows the layout of the MATLAB application, called as Volume, Conductivity and Depolarization Factor for Spheroidal Polarization Tensor (VCD_SPT), when running in a Windows

environment. The application is a standalone software application developed using MATLAB App Designer, with the source code compiled using MATLAB Compiler. Figure 3 (a) is the menu for the fixed or prescribed conductivity, k, where, after inserting the values for M_1, M_2 and k as inputs, the volume |B| and the depolarization factor d_1 are displayed when the button Calculate is clicked.

Meanwhile, Figure 3 (b) is the menu when the volume |B| is fixed or prescribed. Since Newton's method is used, users also need to provide the initial value for k and d_1, together with M_1, M_2 and |B| in order to determine k and d_1. Once all inputs are specified and the push button Calculate is clicked, the values for conductivity k and depolarization factor d_1 will be computed and displayed. Although users need some knowledge to select suitable initial values, the application still simplifies the overall computational process by performing the iterative calculations in Newton's method automatically. The initial values can be estimated from the known properties of the conducting object or selected based on reasonable assumptions and previous experience with similar cases.

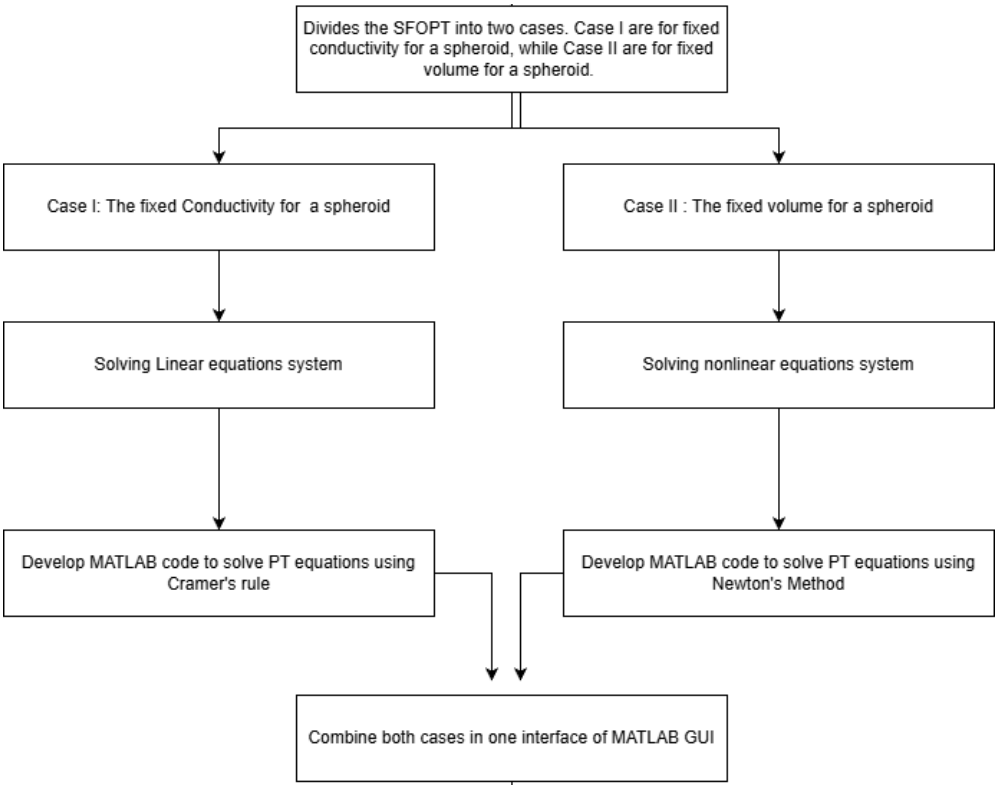


Fig. 2. The related flowchart in preparing the MATLAB Application with two separated menus

4. Examples on Using VCD_SPT

In order to demonstrate the capabilities of VCD_SPT, previous examples from Khairuddin *et al*., [5] and Khairuddin *et al*., [21] will be used. Figure 4 shows the display when the inputs are M_1=0.000032, M_2=0.000042 and k=10000000 resulting the outputs, volume|B|=0.00001268 and depolarization factor d_1=0.3962. The first order PT used in this example represents the pyramid in Khairuddin *et al*., [5], with its length, width, and height all equal to 0.03. In Khairuddin *et al*., [5], at the same conductivity, this pyramid can be represented by an oblate spheroid with a shorter semi-axis of 0.0124, while the other two semi-axes are longer, with a length of 0.0156. This indicates that the application produces a small relative error of around 0.3% in computing the volume of the spheroid. Moreover, the volume of the equivalent spheroid obtained from the application is

1.2678×10−5, compared with 9×10−6 for the volume of the pyramid, giving an absolute difference of approximately 3.64×10−6. The close agreement between the two

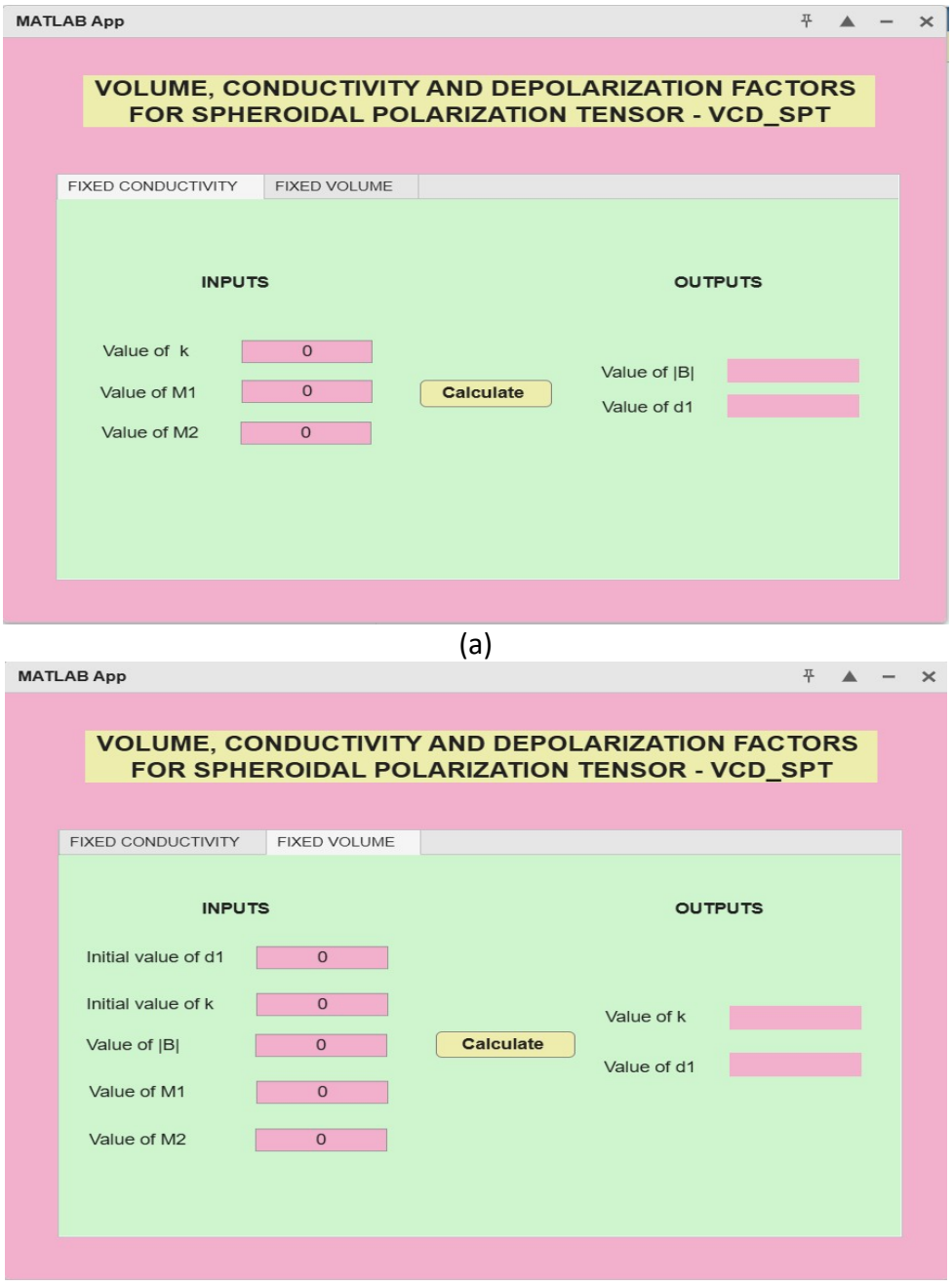


Fig. 3. The layout for the MATLAB application called as Volume, Conductivity and Depolarization Factor for Spheroidal Polarization Tensor (VCD_SPT), when running in a Windows with (a) for fixed conductivity and (b) for fixed volume

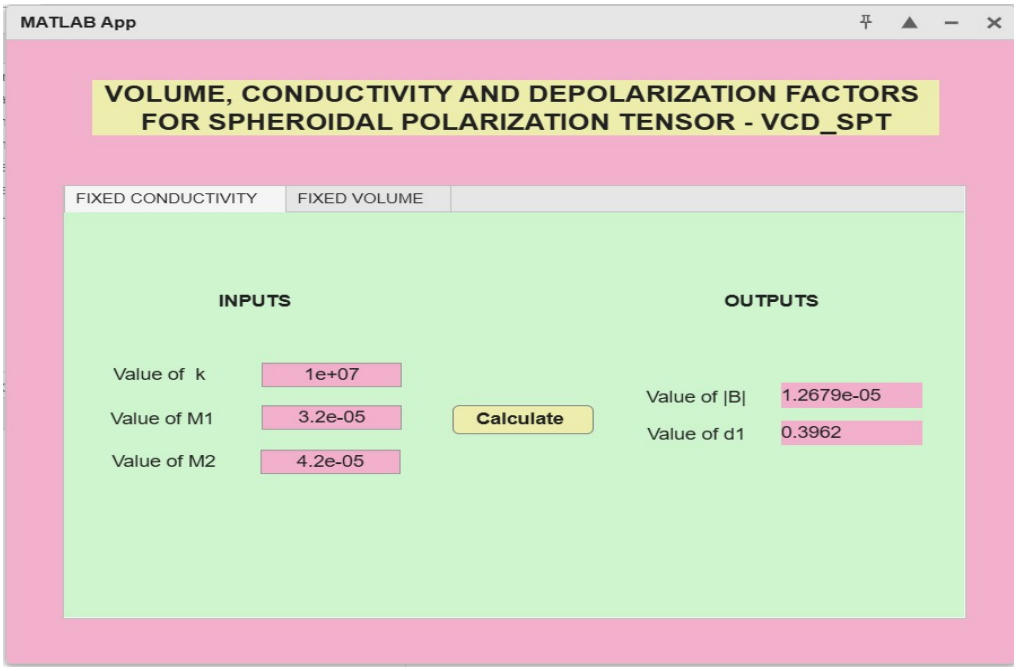


Fig. 4. The display by MATLAB application VCD_SPT when the inputs are M_1=0.000032, M_2=0.000042 and k=10000000 with the resulting outputs |B|=0.00001268 and d_1= 0.3962

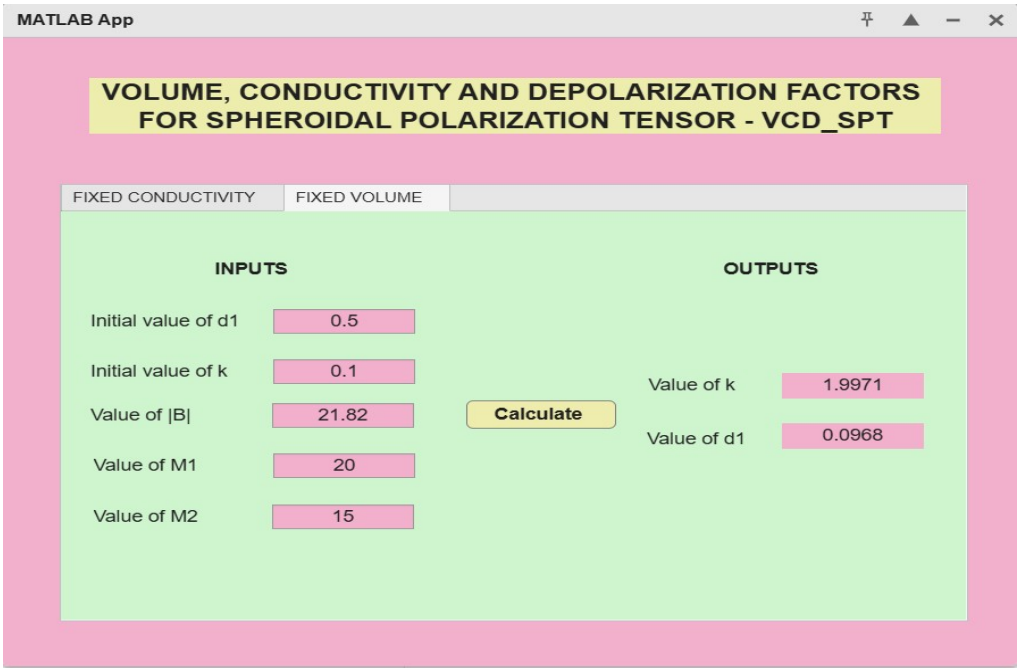


Fig. 5. The outputs k=1.9971 and d_1=0.0968 computed by VCD_SPT with the initial values 0.1 and 0.5, respectively, with the inputs are M_1=20, M_2=15 and |B|=21.82

volumes, as reflected by their absolute difference, demonstrates that the volume of the spheroidal representation provides a useful approximation of the overall size of the pyramid. On the other hand, the value of d_1 might be useful later on for determining the corresponding semi-axes of the spheroid, as shown in Yunos *et al*., [16], which provides additional information about its geometric dimensions.

In addition, Figure 5 shows the display of VCD_SPT with the inputs are M_1=20, M_2=15 and |B|=21.82 and the outputs are k=1.9971 and d_1=0.0968. The first order PT used in this example is

taken from [21] where, in [21], given M_1=20 and M_2=15, k is fixed to 2 for computing |B|=21.8182 and d_1=0.0909. Now, when the volume is fixed to be almost identical to that in Khairuddin *et al*., [21] for computing k and d_1 using VCD_SPT with the chosen initial values, the resulting relative differences in k and d_1, when compared with [21], are 0.2% and 6%, respectively. These small relative differences suggest that the method can reliably obtain the conductivity and depolarization factor of the spheroid. The results also demonstrate that VCD_SPT can reproduce the parameters obtained using the approach in Khairuddin *et al*., [21] when a comparable volume is prescribed.

5. Discussion and Conclusion

During this study, a MATLAB application called VCD_SPT has been developed to facilitate the computation of a few parameters of a spheroid, based on the first order PT, when either the conductivity or the volume is fixed. When the conductivity is fixed, the volume and depolarization factor can be determined, while when the volume is fixed, the conductivity and depolarization factor can be obtained. In industry, the application can support the characterization of conducting objects by providing a simpler representation of their size and electrical properties. Furthermore, the application may also assist in practical applications involving object detection, material characterization, and electric field-based sensing where efficient parameter estimation is required.

In addition, VCD_SPT can also be used to enhance teaching and learning in the higher education institutions for science and engineering students. In the course of Linear Algebra, for the given M_1 and M_2 with the fixed k, the lecturer can ask students to develop a system of linear equations with the unknowns |B| and d_1 before solving the system and verifying the solution with VCD_SPT. Similarly, in the course of Numerical Methods, for the given M_1 and M_2 with the fixed |B|, lecturers can demonstrate the nonlinear equations involved before discussing the solutions with the students. The numerical results can then be checked with VCD_SPT. This approach not only supports technology-enhanced learning but also helps students understand how mathematical formulations can be applied to real-world applications such as object detection, material characterization, and electric field-based sensing.

Finally, VCD_SPT can encourage collaboration among academicians and researchers by creating opportunities to share knowledge and research ideas across different disciplines. Such collaboration can contribute to the Sustainable Development Goals (SDGs), particularly by supporting quality education and fostering partnerships for sustainable development. In the long term, continued collaboration can strengthen knowledge sharing, promote new research opportunities, and contribute to broader academic and societal impact.

References

[1] Ammari, Habib, and Hyeonbae Kang. *Polarization and moment tensors: with applications to inverse problems and effective medium theory*. Springer Science & Business Media, 2007. https://doi.org/10.1007/978-0-387-71566-7

[2] Ammari, Habib, Thomas Boulier, and Josselin Garnier. "Modeling active electrolocation in weakly electric fish." *SIAM Journal on Imaging Sciences* 6, no. 1 (2013): 285-321. https://doi.org/10.1137/12086858X.

[3] Khairuddin, Taufiq KA, and W. R. B. Lionheart. "Does electro-sensing fish use the first order polarization tensor for object characterization? Object discrimination test." *Sains Malaysiana* 43, no. 11 (2014): 1775-1779.

[4] Khairuddin, Taufiq Khairi Ahmad, and William RB Lionheart. "Polarization tensor: Between Biology and engineering." *Malaysian Journal of Mathematical Sciences* 10 (2016): 179-191.

[5] Khairuddin, Taufiq K. Ahmad, and William RB Lionheart. "Characterization of objects by electrosensing fish based on the first order polarization tensor." *Bioinspiration & biomimetics* 11, no. 5 (2016): 055004. https://doi.org/10.1088/1748-3190/11/5/055004.

[6] Lionheart, William R., and Taufiq K. Ahmad Khairuddin. "Polarization tensors and object recognition in weakly electric fish." 2017.
[7] Khairuddin, Taufiq KA, Paul D. Ledger, and William RB Lionheart. "Investigating the polarization tensor to describe and identify metallic objects." In *Proc. of the World Congress on Engineering*, vol. 1, pp. 122-127. 2015.
[8] Marsh, Liam A., Christos Ktistis, Ari Järvi, David W. Armitage, and Anthony J. Peyton. "Determination of the magnetic polarizability tensor and three dimensional object location for multiple objects using a walk-through metal detector." *Measurement Science and Technology* 25, no. 5 (2014): 055107. https://doi.org/10.1088/0957-0233/25/5/055107.
[9] Ledger, P. D., W. R. B. Lionheart, and A. A. S. Amad. "Characterisation of multiple conducting permeable objects in metal detection by polarizability tensors." *Mathematical Methods in the Applied Sciences* 42, no. 3 (2019): 830-860. https://doi.org/10.1002/mma.5387.
[10] Ledger, Paul David, and William RB Lionheart. "Properties of generalized magnetic polarizability tensors." *Mathematical Methods in the Applied Sciences* 46, no. 5 (2023): 5604-5631. https://doi.org/10.1002/mma.8856.
[11] Khairuddin, Taufiq Khairi Ahmad, and William Lionheart. "Some properties of the first order polarization tensor for 3-D domains." *Matematika* (2013): 1-18.
[12] Amad, Alan AS, Paul D. Ledger, Timo Betcke, and Dirk Praetorius. "Benchmark computations for the polarization tensor characterization of small conducting objects." *Applied Mathematical Modelling* 111 (2022): 94-107.https://doi.org/10.1016/j.apm.2022.06.024.
[13] Sukri, Suzarina Ahmed, Taufiq Khairi Ahmad Khairuddin, Mukhiddin Muminov, Yeak Su Hoe, and Syafina Ahmad. "The Effect of Different Scale on Object to the Approximation of the First Order Polarization Tensor of Sphere, Ellipsoid, and Cube." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 87, no. 1 (2021): 108-117. https://doi.org/10.37934/arfmts.87.1.108117.
[14] Ahmad, Syafina, N. Mohamad Yunos, TK Ahmad Khairuddin, Mukhiddin Muminov, and S. Ahmed Sukri. "The Effect on The Volume and Semi Axes of a Conducting Spheroid Due to The Scaling on Its First Order Polarization Tensor." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 94, no. 1 (2022): 1-18. https://doi.org/10.37934/arfmts.94.1.118.
[15] Mohamad Yunos, Nurhazirah, Taufiq Khairi Ahmad Khairuddin, and Nor Muhainiah Mohd Ali. "The Orientation of a Rotated Ellipsoid Based on Its Polarization Tensor." In *International Seminar on Mathematics in Industry*, pp. 83-95. Cham: Springer Nature Switzerland, 2024. https://doi.org/10.1007/978-3-031-85926-7_6.
[16] Yunos, Nurhazirah Mohamad, Taufiq Khairi Ahmad Khairuddin, and Siti Nur Auni Rusli. "The Conductivity and the Depolarization Factor of a Spheroid based on its Polarization Tensor and Volume." In *2025 21st IEEE International Colloquium on Signal Processing & Its Applications (CSPA)*, pp. 331-336. IEEE, 2025. https://doi.org/10.1109/CSPA64953.2025.10932992
[17] Yunos, Nurhazirah Mohamad, Taufiq Khairi Ahmad Khairuddin, Sharidan Shafie, Tahir Ahmad, and William Lionheart. "The depolarization factors for ellipsoids and some of their properties." *Malaysian Journal of Fundamental and Applied Sciences* 15, no. 6 (2019): 784-789. https://doi.org/10.11113/mjfas.v15n6.1364
[18] Lionheart, William RB. "EIT reconstruction algorithms: pitfalls, challenges and recent developments." *Physiological measurement* 25, no. 1 (2004): 125-142. https://doi.org/10.1088/0967-3334/25/1/021
[19] Adler, Andy, and David Holder, eds. *Electrical Impedance Tomography: Methods, History and Applications*. 2nd ed. Boca Raton, FL: CRC Press. 2022. https://doi.org/10.1201/9780429399886
[20] Khairuddin, TK Ahmad, and WILLIAM RB Lionheart. "Fitting ellipsoids to objects by the first order polarization tensor." *Malaya Journal of Matematik* 1, no. 04 (2013): 44-53. https://doi.org/10.26637/mjm104/005
[21] Khairuddin, Taufiq Khairi Ahmad, Nurhazirah Mohamad Yunos, and Sharidan Shafie. "Fitting the first order PT by spheroid: A semi analytical approach." In *AIP Conference Proceedings*, vol. 2184, no. 1, p. 060059. AIP Publishing LLC, 2019. https://doi.org/10.1063/1.5136491
[22] Bahuriddin, Nur Safirah, Mukhiddin Muminov, Taufiq Khairi Ahmad Khairuddin, Syafina Ahmad, and Wan Rohaizad Wan Ibrahim. "An Extended Method for Fitting the First Order Polarization Tensor to a Spheroid." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 23 (2021): 8-17. https://doi.org/10.37934/araset.23.1.817
[23] NURHAZIRAH, MOHAMAD YUNOS, KHAIRI AHMAD KHAIRUDDIN TAUFIQ, MUHAINIAH MOHD ALI NOR, and AHMED SUKRI SUZARINA. "Spheroidal first order polarization tensor (SFOPT) toolkit." *SEMARAK INTERNATIONAL JOURNAL OF FUNDAMENTAL AND APPLIED MATHEMATICS Учредители: Akademia Baru Publishing* 1, no. 1 (2024): 24-37. https://doi.org/10.37934/sijfam.1.1.2437
[24] Abdul Latiff, Nurul Tasnim, Siti Nur Auni Rusli, Nurfarrisha Asnida Khairul Akma, Nurhazirah Mohamad Yunos, Taufiq Khairi Ahmad Khairuddin, and Suzarina Ahmed Sukri. "Spheroidal first order polarization tensor (SFOPT)–fixed volume." (2025): 209-213.