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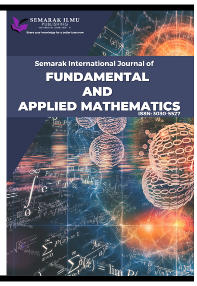

A Piecewise Bezier Surface Interpolation Model for Facial Surface Visualization

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ABSTRACT

Facial surface visualization is a fundamental task in geometric modeling, particularly for applications that require smooth and accurate representation of facial geometry. This paper proposes a piecewise Bézier surface interpolation model for the visualization of facial surfaces constructed from facial landmark data. The proposed approach partitions the facial parametric domain into multiple overlapping patches, where each patch is interpolated independently using a tensor-product Bézier surface formulation. Bernstein polynomial basis functions are employed to compute the surface control points through a least-squares interpolation strategy. A Gaussian blending mechanism is subsequently applied to ensure smooth transitions and geometric continuity between adjacent patches. The resulting composite Bézier surface produces a continuous and visually coherent facial surface while preserving essential geometric characteristics such as curvature variation and boundary structure. Experimental results demonstrate that the proposed model achieves stable and smooth surface visualization even when constructed from sparse facial landmark data. These findings indicate that the proposed piecewise Bézier surface interpolation framework provides an effective and mathematically well-defined solution for facial geometry visualization and computer-aided geometric design applications.

1. Introduction

Facial surface visualization is an essential topic in computer-aided geometric design (CAGD), computer graphics, and computer vision due to its wide range of applications in facial modeling, animation, and geometric analysis [1]. An effective facial surface representation should preserve the intrinsic geometric characteristics of the face while maintaining surface smoothness and continuity across the entire facial region. Conventional facial modeling approaches often rely on polygonal mesh

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representations, which may require dense sampling to achieve visually smooth surfaces and can lead to increased computational complexity when modeling curved facial structures [2].

Parametric surface modeling provides an alternative framework for representing facial geometry in a compact and mathematically well-defined manner. Among various parametric surface representations, Bézier surfaces are well-known for their simplicity, strong geometric properties, and intuitive control through a finite set of control points [3,4]. The mathematical formulation of Bézier surfaces is based on Bernstein polynomials, which guarantee numerical stability and smooth surface construction [4,5]. These properties make Bézier surfaces particularly suitable for modeling smooth organic shapes, such as facial geometry.

Facial landmark detection techniques enable the extraction of meaningful geometric feature points from facial images. Such landmarks provide structured geometric information that can be utilized for surface construction without requiring dense three-dimensional scanning equipment. Landmark-based facial analysis has been extensively studied in the context of facial recognition and animation [6-9]. However, relatively fewer studies emphasize geometric surface visualization using parametric interpolation models derived from facial landmarks.

A single global Bézier surface is often insufficient to accurately represent complex facial geometry due to its limited local control. To address this limitation, piecewise Bézier surface interpolation, also referred to as composite or patch-based Bézier surface modeling, has been introduced in geometric modeling literature to enhance local adaptability while preserving global smoothness [4,10]. By partitioning the parametric domain into multiple patches, local geometric variations can be captured more effectively.

Motivated by these observations, this study proposes a piecewise Bézier surface interpolation model for facial surface visualization using facial landmark data extracted from a frontal facial image. The proposed approach focuses on geometric modeling and visualization rather than facial recognition or biometric classification.

Facial surface visualization is an essential topic in computer-aided geometric design (CAGD), computer graphics, and computer vision due to its applications in facial modeling, animation, and geometric analysis [1]. An effective facial surface representation should preserve the geometric characteristics of the face while maintaining surface smoothness and continuity. Conventional facial modeling techniques often rely on polygonal meshes, which may require dense sampling to achieve smooth surfaces and can lead to increased computational complexity and visual artifacts when representing curved facial structures [2].

Parametric surface modeling provides an alternative framework for representing facial geometry in a compact and mathematically well-defined form. Among various parametric surface representations, Bézier surfaces are widely recognized for their simplicity, smoothness, and strong geometric properties, including affine invariance, convex hull containment, and intuitive control through a finite set of control points, as discussed by Bézier *et al.*, [3] and Farin *et al.*, [4]. These properties make Bézier surfaces particularly suitable for modeling smooth organic shapes, such as facial surfaces.

The mathematical formulation of Bézier surfaces is based on Bernstein polynomials, which guarantee numerical stability and smooth surface construction [4,5]. Due to these advantages, Bézier surfaces and related spline-based surfaces have been extensively applied in geometric modeling, industrial design, and surface visualization [10-12].

The remainder of this paper is organized as follows. Section 2 describes the proposed piecewise Bézier surface interpolation methodology. Section 3 presents the experimental results and visualization analysis. Section 4 concludes the paper and outlines potential directions for future research.

2. Methodology

This section describes the methodology employed to construct the Bézier surface interpolation model for facial surface visualization. The overall process consists of facial landmark extraction, surface parameterization, Bézier surface formulation, and control point determination.

Definition 1 [3], [4]: A tensor-product Bézier surface of degree $p \times q$ is defined as

$$S(u,v) = \sum_{i=0}^{p}\sum_{j=0}^{q} P_{i,j}\beta_i^p(u)\beta_j^q(v), \quad u,v \in [0,1] \tag{1}$$

Where P_{ij} represents the control points of the surface, and $\beta_i^P(u)$ and $\beta_j^q(v)$ are Bernstein basis polynomials of degree p and q respectively.

Definition 2 [5], [10]: The Bernstein basis polynomial is defined as

$$\beta_i^p(u) = \binom{p}{i} u^i (1-u)^{p-i}, \quad i = 0,1,..,p \tag{2}$$

In this study, facial landmark points extracted from a frontal facial image are used as geometric input data. The landmark points are mapped into the parametric domain $[0,1]\times[0,1]$ using coordinate normalization. Separate Bézier surfaces are constructed for the spatial coordinates using a least-squares fitting approach to determine the control points of the surface [5,10].

Definition 2 [5]: The least-squares formulation for estimating the Bézier control points is given by

$$P = \left(B^T B\right)^{-1} B^T D \tag{3}$$

Where B denotes the Bernstein basis matrix is constructed from the parametric coordinates and D represents the facial landmark data. To ensure smooth transitions between adjacent patches, a Gaussian-based blending strategy is applied. The final facial surface is obtained by computing a weighted average of overlapping Bézier patches, resulting in a continuous and visually coherent composite surface.

3. Results and Discussion

The proposed piecewise Bézier surface interpolation model was evaluated using facial landmark data extracted from a frontal facial image.

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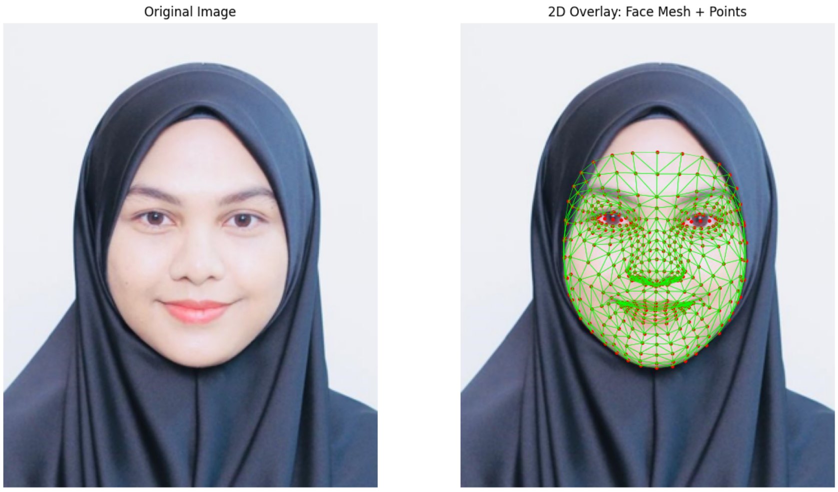

Fig. 1. 2D facial landmark detection and mesh overlay using MediaPipe Face Mesh

Figure 1 presents the 2D facial landmark detection results obtained from a frontal facial image using the MediaPipe Face Mesh framework. The original input image is shown on the left, while the right image illustrates the detected facial structure overlaid with landmark points and mesh connections. A total of 468 facial landmarks is identified and distributed uniformly across key facial regions, including the eyes, eyebrows, nose, mouth, jawline, and forehead.

Based on the extracted 2D facial landmarks shown in Figure 1, the detected points were subsequently transformed into a three-dimensional Cartesian coordinate system to analyse their spatial distribution.

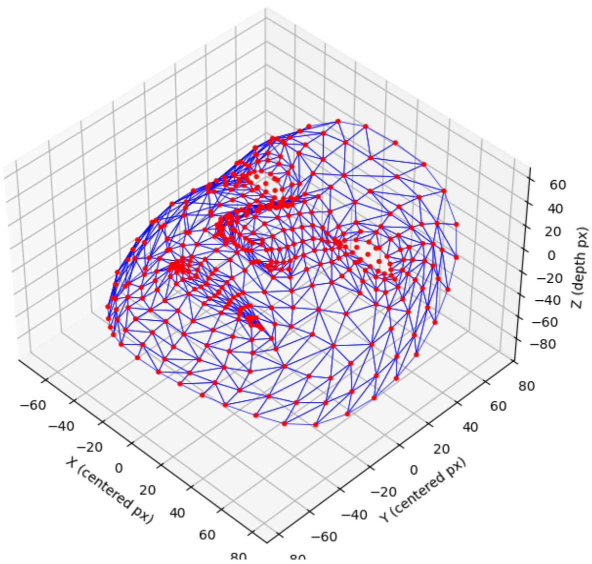

Fig. 2. 3D Facial landmark distribution

Figure 2 illustrates the three-dimensional distribution of the detected facial landmarks in a centered Cartesian coordinate system. Despite the use of sparse landmark data, essential facial structures such as the forehead, eye regions, nose bridge, and mouth contour are clearly identifiable. This observation confirms that facial landmark data provide sufficient geometric information for parametric surface interpolation. The consistency observed between the 2D landmark overlay and the 3D spatial distribution validates the effectiveness of the facial landmark extraction process and justifies its use as input data for piecewise Bézier surface interpolation.

Based on the landmark distribution, the facial parametric domain was partitioned into multiple overlapping patches. A tensor-product Bézier surface was independently interpolated within each patch using the least-squares control point estimation approach described in Section 2.

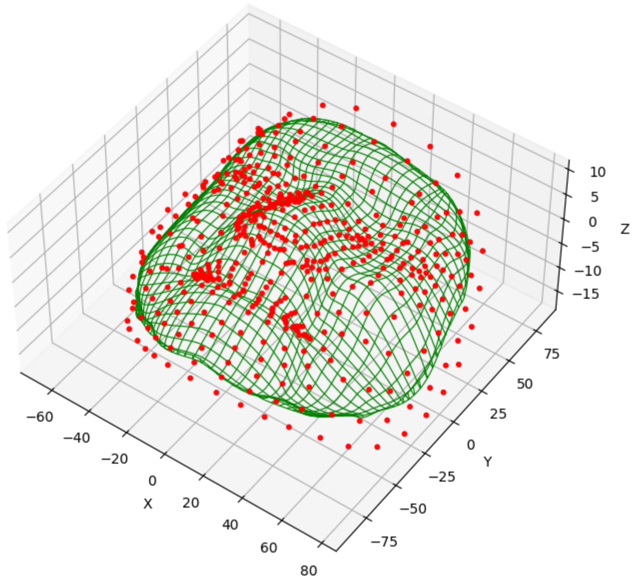

Fig. 3. Piecewise Bézier surface interpolation with structured net lines

Figure 3 presents the resulting piecewise Bézier surface interpolation, where structured net lines are overlaid to visualize the underlying parametric organization of the surface. The interpolated surface exhibits smooth curvature transitions across patch boundaries, indicating that the blending strategy effectively ensures geometric continuity. Regions with higher curvature, such as the nose and eye areas, are accurately captured, demonstrating the adaptability of the patch-based Bézier formulation to local geometric variations. The visualization results show that the interpolated surface provides a continuous and visually coherent facial surface representation, consistent with the geometric properties of Bézier surfaces discussed in Farin [4], and Piegl and Tiller [10]. Quantitative evaluation was conducted using the root mean square error (RMSE), defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left\| S\left(u_k v_k\right) - D_k \right\|^2} \quad (4)$$

Where $S\left(u_k, v_k\right)$ denotes the fitted Bézier surface point and D_k is the corresponding facial landmark point. The RMSE results indicate that the fitted Bézier surface closely approximates the landmark data, confirming the stability and accuracy of the interpolation process. Compared to mesh-based facial representations [2], the proposed approach offers a compact parametric surface model with fewer parameters while maintaining surface smoothness, as similarly observed in spline-based modeling studies [9,13-16].

4. Conclusions

This paper presented a piecewise Bézier surface interpolation model for facial surface visualization based on facial landmark data. By partitioning the facial parametric domain into multiple overlapping patches, the proposed approach enhances local geometric adaptability while maintaining global surface smoothness.

The experimental results demonstrate that Bézier control points derived directly from facial landmarks are sufficient to capture essential facial features, including boundary contours and curvature variations. The application of a blending strategy ensures smooth transitions between adjacent patches, resulting in a continuous and visually coherent facial surface.

The visualization outcomes confirm that the proposed model can effectively reconstruct smooth facial surfaces even when constructed from sparse landmark data. This highlights the potential of the proposed piecewise Bézier surface interpolation framework as a compact and mathematically well-defined solution for facial geometry visualization and computer-aided geometric design applications.

Future work may explore adaptive patch partitioning strategies, feature-driven refinement near critical facial regions, and integration with uncertainty-aware or fuzzy geometric modelling frameworks.

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