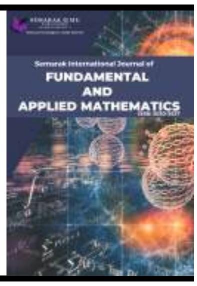
SEMARAK ILMU PUBLISHING

Semarak International Journal of Fundamental and Applied Mathematics

Journal homepage: https://semarakilmu.my/index.php/sijfam
ISSN: 3030-5527



Unique Solution of Quartic Diophantine Equation $x^4 - (n^2 + 4)y^4 = 1$

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ARTICLE INFO

Article history:
Received 20 December 2025
Received in revised form 25 February 2026
Accepted 10 April 2026
Available online 11 May 2026

Keywords:
Diophantine equation; quartic equation; square-free integer; positive integer solution; number theory

ABSTRACT

Diophantine equations play an important role in number theory due to their connections with integer solutions and classical structures. Among these, quartic Diophantine equations $x^4 - Dy^4 = 1$, where D is a square-free integer, are known for their complexity. In this research, we investigate the parametric family of quartic Diophantine equation $x^4 - (n^2 + 4)y^4 = 1$, where n is a positive integer and $n^2 + 4$ is square-free. The significance of this research lies in unifying the isolated cases of D for $x^4 - (n^2 + 4)y^4 = 1$ in a single-structured parametric family. The purpose of this work is to determine whether positive integer solutions exist for this entire family and, if so, to characterize the solution explicitly. In order to achieve this, the odd and even cases of n are investigated independently and formulated as propositions. We show that the equation has no positive solutions for $n \geq 2$. Moreover, we prove that the equation has a unique positive integer solution, namely $(n, x, y) = (1,3,2)$.

1. Introduction

A Diophantine equation is an equation with integer coefficients whose solutions are constrained to integers [1]. Among the most famous examples is

$$x^n + y^n = z^n \,, \tag{1}$$

where Pierre de Fermat asserted that **Eq. (1)** has no non-trivial integer solution for any integer $n > 2$, a claim later known as Fermat's Last Theorem [2]. A proof to **Eq. (1)** became an unsolved problem that sparked centuries of mathematical innovation, until Andrew Wiles presented a complete proof in the late 20th century [3]. More broadly, Fermat's work helped establish Diophantine equations as central objects within number theory.

Today, Diophantine equation is widely applied in real life. In chemistry, the concept of integer solution plays an important role in balancing chemical reactions, where coefficients must be whole numbers satisfying conservation laws that can be represented by an implicit linear Diophantine

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equation [4]. In addition, the molecular formula of organic compounds, with given molecular weight, can be reformulated to become a problem of Diophantine equation [5]. Moreover, Diophantine equation has become one of the important domains in cryptography. A new secure authentication method is introduced that leverages positive integer solutions of quadratic Diophantine equations, where this approach can be applied to generate one-time passwords or electronic tokens for use in cryptographic systems [6]. Furthermore, a mathematical framework for an information security system based on a linear inhomogeneous Diophantine equation has been introduced, together with the procedures for determining the solution of the Diophantine equation [7]. A framework for multivariate public-key cryptosystems that integrates concepts from both triangular constructions and oil-vinegar schemes has also been presented, in which a new public-key cryptosystem that relies on solving a Diophantine equation defined over polynomial rings is proposed [8].

Numerous real-world applications of Diophantine equations across science and technology emphasize the importance to investigate different structural families within this context. As these equations grow in complexity, their algebraic behavior often reveals arithmetic patterns and connections to modern number theory. A notable example comes from the study of quartic Diophantine equations, particularly of the form

$$x^4 - Dy^4 = 1, \tag{2}$$

where D is a square-free integer. **Eq. (2)** presents a unique class of Diophantine equations involving square-free integers. We focus on a specific parametric family given by

$$x^4 - (n^2 + 4)y^4 = 1, \tag{3}$$

where n is a positive integer and $n^2 + 4$ is square-free.

The objective of this research is to establish the positive non-trivial integer solution of **Eq. (2)**, in order to address the existing research gap concerning the absence of a unified structural classification for quartic Diophantine equations of this form. The significance of this research lies in unifying many of the previously isolated cases of quartic Diophantine equations. Each isolated case corresponds to a fixed value of the parameter D in **Eq. (2)** which can be studied in a single-structured framework, thereby demonstrating that an entire family of such equations can be completely classified rather than handled individually.

This paper is structured as follows: Section 2 presents relevant literature and Section 3 describes the research methodology of this work. The results and discussion of the study are presented in Section 4, while the findings are concluded in Section 5.

2. Literature Review

Diophantine equations can have multiple solutions or none at all, depending on their structure and the integer coefficients [9]. This complexity outlines the rich landscape of Diophantine equations and the challenges in finding integer solutions. Literature review on Diophantine equations with unique solution is first presented in the following subsection.

2.1 Unique Solution of Diophantine Equations

The investigation of unique solutions to Diophantine equations has emerged as one of the most active areas within number theory. Panda [10] determined that the non-linear Diophantine equation

$$8x^4 + 1 = y^2 \tag{4}$$

has a unique non-trivial positive integer solution $(x, y) = (1, 3)$. Moreover, Aggarwal and Upadhyaya [11] showed that the equation

$$8\alpha + 67\beta = \gamma^2 \tag{5}$$

has a unique solution in non-negative integers, which is $(\alpha, \beta, \gamma) = (1, 0, 3)$, with the aid of Catalan's conjecture. For the Diophantine equation with prime number coefficients, Tadee [12] demonstrated that the Diophantine equation $(p + 6)x - py = z^2$ has unique non-negative integer solution $(0, 0, 0)$ for prime p with the constrain $p \equiv 1 \pmod{28}$.

Besides that, Diophantine equations in which either the base, the exponent, or both are unknown, are called exponential Diophantine equations [13], which display highly restrictive solution structures. In many cases, the existence of even a single integer solution is remarkable. Chuayjan et al., [14] established the unique solution in non-negative integer solution $(x, y, z) = (0, 0, 0)$ for the equation

$$15^x - 17^y = z^2. \tag{6}$$

In addition, Thongnak *et al.*, [15] obtained the unique solution $(0, 0, 0)$ for the equation

$$7^x - 5^y = z^2, \tag{7}$$

by applying Catalan's conjecture and some congruence-related theorems. Borah and Dutta [16] investigated on a specific family of exponential Diophantine equation

$$n^x + 24^y = z^2, \tag{8}$$

for $n \equiv 5$ or $7 \pmod 8$. The results include the unique positive integral solution $(x, y, z) = (2, 1, 7)$ for the equation $5^x + 24^y = z^2$ and unique non-negative integer solution $(x, y, z) = (0, 1, 5)$ for $n \equiv 5$ or $7 \pmod 8$ in **Eq. (8)**, provided that $n + 1$ is square free. In the case where $n + 1$ is a perfect square, **Eq. (8)** has non-negative integer solutions of (x, y, z), which are $(0, 1, 5)$ and $(1, 0, \sqrt{n + 1})$.

Following this, the literature review on Diophantine equations with square-free integer is discussed.

2.2 Diophantine Equations with Square-Free Integer

A further important branch of study is the Diophantine equation involving square-free integers that presents a setting with arithmetic properties. One of the most classical examples is Pell's equation [9]:

$$x^2 - Dy^2 = 1, \tag{9}$$

where D is a positive square-free integer, which has infinitely many integer solutions. Consequently, finding the fundamental solution, i.e. the smallest positive integer solution of **Eq. (9)**, has become a topic of particular interest [7].

Additionally, Ljunggren [17] proved that the Diophantine equation

$$x^2 - Dy^4 = 1, \tag{10}$$

where D is a positive square-free integer, has at most two positive integer solutions. Furthermore, he extended his work to the Diophantine equation

$$x^2 + 1 = 2y^4, \tag{11}$$

where the positive integer solutions of (x, y) are $(1, 1)$ and $(239, 13)$. Due to the complicated proof presented by Ljunggren, some researches simplify the proof or provide alternative approaches to determine the positive integer solutions of **Eq. (11)** [18-21].

3. Methodology

In order to determine the positive integer solutions of **Eq. (3)**, the equation is analyzed corresponding to the cases where n is even and odd, thereby allowing the structure of the equation to be examined in each setting. In this framework, we show that no positive integer solution exists for $n \geq 2$ by using congruence arguments and algebraic factorizations aided with known results in **Eq. (4)** and **Eq. (11)**. The case study methodology exhausts all possible cases for $n \geq 2$, leaving the remaining case $n = 1$, for which the positive integer solution of **Eq. (3)** is explicitly obtained.

4. Result and Discussion

We first begin by observing **Eq. (3)** has trivial integer solutions $(\pm 1, 0)$ based on the structure of the equation. Consequently, these solutions are excluded from further consideration. The focus of this section is therefore placed on the positive integer solutions, which constitute the interest of this research.

We first consider the case where n is even, which is addressed in Proposition 1.

Proposition 1. Let n be a positive even integer. Then the Diophantine equation

$$x^4 - (n^2 + 4)y^4 = 1$$

has no positive integer solution.

Proof. Given that n is even, we consider $n = 2m$ for $m \in \mathbb{N}$. Then the Diophantine equation becomes

$$x^4 - 4(m^2 + 1)y^4 = 1, \tag{12}$$

where $m^2 + 1$ is a positive square-free integer, since $n^2 + 4$ is square-free. Based on **Eq. (12)**, we infer that x^4 is odd, which follows that x^2 is odd and x is odd. Then **Eq. (12)** can be written in the form:

$$(x^2-1)(x^2+1)=4(m^2+1)y^4. \quad (13)$$

Since x^2 is odd, then it follows that x^2-1 and x^2+1 are even. Let $x^2-1=2a$ for $a\in\mathbb{N}$, it follows that $x^2+1=2(a+1)$, then **Eq. (13)** can be expressed as

$$a(a+1)=(m^2+1)y^4. \quad (14)$$

Since $\gcd(a,a+1)=1$ and y^4 is an integer of fourth power, the factors of m^2+1 and y^4 are distributed entirely in a and $a+1$.

Suppose $a=u^4$ and $a+1=(m^2+1)v^4$, where $\gcd(u,v)=1$ and $y=uv$. This leads to $x^2-1=2u^4$ and $x^2+1=2(m^2+1)v^4$, which yields

$$(m^2+1)v^4-u^4=1. \quad (15)$$

Furthermore, $x^2=2u^4+1$ implies that $2u^4+1$ is a perfect square. Since x is odd, we consider $x=2k+1$, where $k=0,1,2,\ldots$, and obtain

$$(2k+1)^2=2u^4+1, \quad (16)$$

which yields $u^4=2k(k+1)$. Hence, we can deduce that u^4 is even, and consequently u is even. Let $u=2j$ for $j\in\mathbb{N}$, we obtain

$$k(k+1)=8j^4. \quad (17)$$

Since k and $k+1$ are consecutive integers, then either one of k or $k+1$ is even, while the other is odd.

We claim that k is odd and $k+1$ is even, then $k=b^4$ and $k+1=8c^4$ for $b,c\in\mathbb{N}$ such that $\gcd(b,c)=1$ and $bc=j$. Using the fact that

$$b^4\equiv 0 \text{ or } 1 \pmod 4, \quad (18)$$

then $b^4+1\equiv 1 \text{ or } 2 \pmod 4$. However, this contradicts with $k+1=b^4+1=8c^4\equiv 0 \pmod 4$. Therefore, the claim is invalid.

Next, we claim that k is even and $k+1$ is odd, then $k=8c^4$ and $k+1=b^4$ for $b,c\in\mathbb{N}$ such that $\gcd(b,c)=1$ and $bc=j$. This yields another Diophantine equation

$$8c^4+1=b^4. \quad (19)$$

Let $d=b^2\in\mathbb{N}$, then

$$8c^4+1=d^2. \quad (20)$$

Eq. (20) is exactly similar to **Eq. (4)**, where there is a unique positive integer solution $(c,d)=(1,3)$. However, $d=b^2=3$ implies that $b\notin\mathbb{N}$, which is a contradiction. Therefore, the claim is again invalid.

Both claims are invalid, leading to the assumption that $a = u^4$ and $a + 1 = (m^2 + 1)v^4$ is false. Then, suppose $a = (m^2 + 1)v^4$ and $a + 1 = u^4$ with $\gcd(u, v) = 1$ and $y = uv$, which follows that $x^2 - 1 = 2(m^2 + 1)v^2$ and $x^2 + 1 = 2u^4$. Consequently,

$$u^4 - (m^2 + 1)v^4 = 1. \quad (21)$$

The equation

$$x^2 + 1 = 2u^4 \quad (22)$$

is exactly similar to **Eq. (11)**, where the positive integer solutions of (x, u) are $(1,1)$ and $(239,13)$. When $u = 1$, then $(m^2 + 1)v^4 = 0$ based on **Eq. (21)**. This implies that $v = 0$ since $m^2 + 1$ is positive, which yields $y = 0$, a trivial solution. When $u = 13$, then

$$(m^2 + 1)v^4 = 13^4 - 1 = 28560 = 2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 17. \quad (23)$$

Since v^4 is an integer of fourth power, so the possible values of v^4 are the divisors of 28560 that are of fourth power, namely 1^4 and 2^4. However, $v = 1$ implies that $m^2 + 1 = 28560$ while $v = 2$ implies that $m^2 + 1 = 1785$. Both values of v lead to $m \notin \mathbb{N}$, which is a contradiction. Therefore, the assumption that $a = (m^2 + 1)v^4$ and $a + 1 = u^4$ is false.

Both possible factorizations of **Eq. (14)** therefore lead to contradictions. Hence **Eq. (12)** and the original equation, i.e. **Eq. (3)**, has no positive integer solution when n is a positive even integer. ■

We now proceed to the case where n is odd, which is treated separately in the following proposition.

Proposition 2. Let n be an odd integer where $n \geq 3$. Then the Diophantine equation

$$x^4 - (n^2 + 4)y^4 = 1$$

has no positive integer solution.

Proof. For odd $n \geq 3$, we consider $n = 2m + 1$ for $m \in \mathbb{N}$, then the Diophantine equation becomes

$$x^4 - (4m^2 + 4m + 5)y^4 = 1. \quad (24)$$

If x is even and y is odd, then $x^4 \equiv 0 \pmod 8$ and $y^4 \equiv 1 \pmod 8$. However,

$$4m^2 + 4m + 5 = 4m(m + 1) + 5 \equiv 5 \pmod 8, \quad (25)$$

since one of the consecutive integers m and $m + 1$ is even. This yields $x^4 \equiv 6 \pmod 8$ based on **Eq. (24)** and leads to a contradiction. Moreover, x is even and y is even are not possible, since the difference between two even integers is impossible to be 1 (see **Eq. (24)**).

Therefore, we conclude that x is odd, which implies that x^2 is odd. It follows that $x^2 - 1$ and $x^2 + 1$ are even. We first express **Eq. (24)** into

$$(x^2 - 1)(x^2 + 1) = (4m^2 + 4m + 5)y^4, \quad (26)$$

and deduce that y^4 is even, consequently y is even. Let $x^2 - 1 = 2a$ and $y = 2b$ for $a, b \in \mathbb{N}$, then **Eq. (26)** becomes

$$a(a+1) = 4(4m^2 + 4m + 5)b^4. \tag{27}$$

Moreover, the odd perfect square x^2 is congruent to $1 (\mathrm{mod}\ 8)$, which leads to $2a + 1 \equiv 1\ (\mathrm{mod}\ 8)$ and subsequently $a \equiv 0\ (\mathrm{mod}\ 4)$. Hence, a is even and divisible by 4, and it follows that $a + 1$ is odd. From **Eq. (27)**, since a and $a + 1$ are consecutive integers, then $\gcd(a, a + 1) = 1$, and b^4 is an integer of fourth power, hence the factors of $4m^2 + 4m + 5$ and b^4 are distributed entirely in a and $a + 1$.

Suppose $a = 4(4m^2 + 4m + 5)c^4$ and $a + 1 = d^4$ for $c, d \in \mathbb{N}$ where $\gcd(c, d) = 1$ and $b = cd$. This leads to

$$4(4m^2 + 4m + 5)c^4 + 1 = d^4\ , \tag{28}$$

which yields

$$x^2 = 2a + 1 = 8(4m^2 + 4m + 5)c^4 + 1 \tag{29}$$

and

$$x^2 = 2d^4 - 1, \tag{30}$$

which is a similar form of **Eq. (11)**. The positive integer solutions of (x, d) are $(1,1)$ and $(239,13)$. When $d = 1$, then $4(4m^2 + 4m + 5)c^4 = 0$ from **Eq. (28)**. It follows that $c^4 = 0$ and $c = 0$ since $4(4m^2 + 4m + 5)$ is positive. Consequently, $b = cd = 0$ and $y = 2b = 0$, a trivial solution. When $d = 13$, based on **Eq. (28)**, we obtain

$$(4m^2 + 4m + 5)c^4 = 7140 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 17. \tag{31}$$

Since c^4 is an integer of fourth power, then the possible value of c^4 is the divisor of 7140 with fourth power, that is 1^4, which leads to $c = 1$. However, $c = 1$ implies that $4m^2 + 4m + 5 = 7140$, which contradicts with the fact that $4m^2 + 4m + 5$ is odd. Therefore, the assumption that $a = 4(4m^2 + 4m + 5)c^4$ and $a + 1 = d^4$ is false.

Then, suppose $a = 4c^4$ and $a + 1 = (4m^2 + 4m + 5)d^4$ for $c, d \in \mathbb{N}$ where $\gcd(c, d) = 1$ and $b = cd$. This leads to

$$4c^4 + 1 = (4m^2 + 4m + 5)d^4, \tag{32}$$

and

$$x^2 = 2a + 1 = 8c^4 + 1. \tag{33}$$

Eq. (33) is exactly similar to **Eq. (4)**, with the unique positive integer solution $(x, c) = (3, 1)$. Hence, from **Eq. (32)**, $c = 1$ implies $(4m^2 + 4m + 5)d^4 = 5$, and we obtain

$$d^4 = \frac{5}{4m^2 + 4m + 5} \tag{34}$$

However, $4m^2 + 4m + 5 > 5$ for $m \in \mathbb{N}$ which leads to $d^4 < 1$, consequently $d \notin \mathbb{N}$, which is a contradiction. Then the assumption that $a = 4c^4$ and $a + 1 = (4m^2 + 4m + 5)d^4$ is false.

Both possible factorizations of **Eq. (27)** therefore lead to contradictions. Hence **Eq. (24)**, and thus the original equation, i.e. **Eq. (3)**, has no positive integer solution for odd $n \geq 3$. ■

With Proposition 1 and Proposition 2 established, all cases for $n \geq 2$ have been exhausted, and Eq. (3) has no positive integer solution for $n \geq 2$. Consequently, the only remaining possibility occurs when $n = 1$. The corresponding positive integer solution in this case is determined in the following theorem.

Theorem 1. The Diophantine equation

$$x^4 - (n^2 + 4)y^4 = 1$$

has a unique positive integer solution $(n, x, y) = (1,3,2)$.

Proof. By Proposition 1 and Proposition 2, the equation has no positive integer solution for all $n \geq 2$. Therefore, it suffices to consider the remaining case $n = 1$, for which the equation reduces to

$$x^4 - 5y^4 = 1, \quad (35)$$

or similarly

$$(x^2 - 1)(x^2 + 1) = 5y^4. \quad (36)$$

By applying the same reasoning as in the proof of Proposition 2, we obtain that x is odd and y is even. Let $x^2 - 1 = 2a$ and $y = 2b$ for $a, b \in \mathbb{N}$, then **Eq. (36)** can be expressed as

$$a(a + 1) = 20b^4. \quad (37)$$

The same arguments in Proposition 2 determine that a is even and divisible by 4. Since a and $a + 1$ are consecutive integers, then $\gcd(a, a + 1) = 1$, and b^4 is an integer with fourth power, leading to the factors of 20 and b^4 being distributed entirely in a and $a + 1$.

Suppose $a = 20c^4$ and $a + 1 = d^4$ for $c, d \in \mathbb{N}$ where $\gcd(c, d) = 1$ and $b = cd$. We infer that

$$20c^4 + 1 = d^4. \quad (38)$$

However, a is divisible by 4, which leads to $a + 1 = 20c^4 + 1 \equiv 1 \pmod 4$. Then $c = 0$ based on **Eq. (38)**, hence $a = 20c^4 = 0$, which is a contradiction to $a \in \mathbb{N}$.

Suppose $a = 4c^4$ and $a + 1 = 5d^4$ for $c, d \in \mathbb{N}$ where $\gcd(c, d) = 1$ and $b = cd$. It follows that

$$4c^4 + 1 = 5d^4 \quad (39)$$

and

$$x^2 = 2a + 1 = 8c^4 + 1, \quad (40)$$

takes the similar form of **Eq. (4)** and the positive integer solution is given by $(x, c) = (3,1)$. Consequently, $c = 1$ leads to $a = 4c^4 = 4$, implies $d^4 = 1$ based on **Eq. (38)**. Hence, $b = cd = 1$ and $y = 2b = 2$, and **Eq. (34)** has the positive integer solution $(x, y) = (3,2)$.

Therefore, the Diophantine equation has the unique positive integer solution $(n, x, y) = (1,3,2)$, which completes the proof. ■

5. Conclusion

In this paper, we investigated the quartic Diophantine equation

$$x^4 - (n^2 + 4)y^4 = 1,$$

where $n \in \mathbb{N}$ such that $n^2 + 4$ is square-free. The even case of n is presented in Proposition 1, whereas the odd case of $n \geq 3$ is presented in Proposition 2. Based on the propositions, we established that the equation has no positive integer solution for $n \geq 2$. As a consequence, the Diophantine equation is reduced to the case $n = 1$. We showed that this case has a positive integer solution, namely $(x, y) = (3, 2)$. This concludes that the given family of quartic Diophantine equation has the unique positive integer solution $(n, x, y) = (1, 3, 2)$. This study contributes to the understanding of higher-degree Diophantine equation by showcasing the unique solution of a specific parametric family of the equation.

Acknowledgement
The first author would like to acknowledge Kementerian Pendidikan Tinggi (KPT) for MyBrainSc Scholarship 2025.

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