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Unsupervised Logic Mining in Classifying Divorce Outcome

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ABSTRACT

In the current era, the increasing number of divorce cases highlights the critical need to understand the behavioural causes that lead to marital breakdown. This study introduces an enhanced logic mining model developed to classify marital outcomes based on behavioural attributes from the Divorce Prediction dataset. The proposed framework integrates a satisfiability-based reverse analysis model with a neural network structure to extract interpretable logical rules from data. A similarity-based selection method is applied during preprocessing to group related attributes, improving the quality of induced logic. The model identifies optimal attribute combinations and generates clear, human-understandable rules that explain marital stability or risk of divorce. The proposed model was evaluated on the real-world Divorce Dataset composed of psychological and behavioural indicators. The results show that the proposed framework effectively improves logic interpretability and classification performance compared to existing methods based on performance metrices. This study contributes to the development of explainable logic-based models that can support counsellors, researchers, and policymakers in understanding behavioural patterns associated with divorce outcomes. However, the model has been tested only on a single behavioural dataset, which may limit its generalizability. Future research could extend this approach to other social and psychological domains to further validate its performance.

1. Introduction
1.1 Introduction to Divorce

The term “divorce” originates from the Latin word *divotium*, implying separation. It evolved from the terms *divort* or *divortere*, with the prefix “*Di*” representing separation and “*vertere*” meaning to turn in various direction. Based on Prince and McKenry, [1] Divorce also refers to the legal dissolution of a marriage by a court or other competent body and it is often the result of unresolved personal, emotional or relational issues between couples. While marriage is intended to be a lifelong commitment, many relationships face challenges that ultimately lead to separation. Divorce not only

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affects the individuals involved but also has wider emotional, social and economic consequences on children, families and communities.

In recent years, divorce rates have continued to rise indicating a significant shift in social dynamics and relationship expectations. According to the World Population Review (2024), the global average divorce rate is around 1.8 per 1,000 people with certain countries like Russia, United States, and South Korea showing notably high rates. For example, about 45% of first marriages in the U.S. end in divorce. The contributing factors often include lack of communication, emotional disconnect, financial strain, and infidelity.

In Malaysia, divorce has become a pressing social issue. Based on data from the Department of Statistics Malaysia (2023), a total of 62,890 divorce cases were recorded in 2022, showing a 43.1% increase from the previous year (43,936 cases in 2021) as Figure 1. This surge highlights a growing trend of marital breakdowns in both urban and rural populations. The increasing divorce rate not only raises concerns among policy-makers and social workers but also stresses the need for deeper understanding and early intervention strategies.

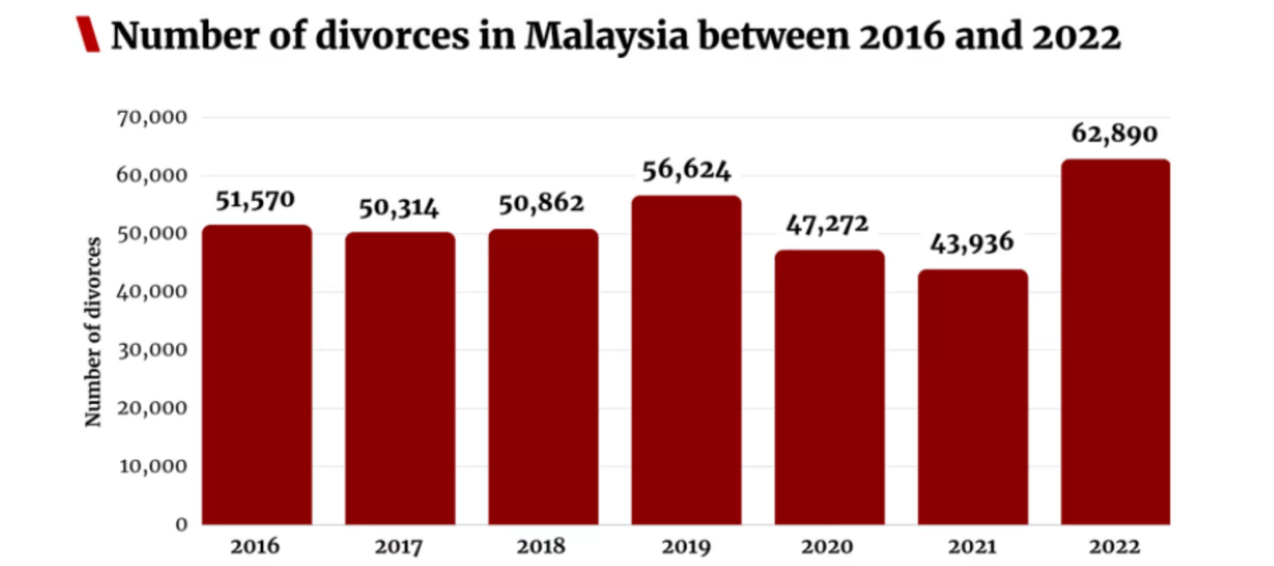

Fig. 1 Divorce cases in Malaysia from 2016 until 2022 (Source: Department of Statistic Malaysia, [2])

Given these concerning trends, there is a clear motivation to explore divorce from a data-driven perspective. Traditional studies often rely on qualitative methods, but modern computational techniques allow us to analyse large datasets and extract meaningful patterns. One area of growing interest is logic mining, which uses symbolic reasoning to uncover interpretable rules from structured data. Such approaches are particularly useful in sensitive domains like family relationships, where transparency and clarity in the results are essential.

To address this, the present study proposes a logic mining approach using the Discrete Hopfield Neural Network (DHNN) on the Divorce Prediction Dataset, which contains 54 binary features based on couples' behaviours and communication patterns. This model aims to extract clear logical rules that can predict the likelihood of divorce based on respondent answers. By identifying key behavioural indicators, the model could support efforts in marriage counselling, social work, and public policy to reduce divorce rates and strengthen family structures.

1.2 Introduction to Logic Mining

In the current era, Artificial Intelligence (AI) models have gained significant attention due to their ability to simulate human intelligence in machines. Among these models, Artificial Neural Networks (ANN) have proven particularly effective for solving complex optimization problems. Inspired by the function neural systems in the human brain, ANN are designed to perform tasks such as classification and pattern recognition within high dimensional datasets. Based on Lamjiak *et al.,* [3] the networks are composed of artificial neurons interconnected units organized into multiple layers typically including input and output layers. These neurons communicate via synaptic weights, which are fundamental in illustrating the strength and connectivity of relationships across the network. These weights are established through training and are retained in the associative memory of the ANN for future retrieval during testing phases.

One notable variant of ANN is the DHNN, introduced by Hopfield and Tank [4]. DHNN is a fully connected recurrent neural network that operates without hidden layers and consists of input and output neurons. Always et al, [5] state that each neuron in the DHNN can be represented in binary form $\{0, 1\}$ or bipolar form $\{-1, 1\}$. To model and analyse the network's behaviour, the Lyapunov energy function is widely used, which aids in expressing and solving computational problems. Figure 2 show the structure of DHNN.

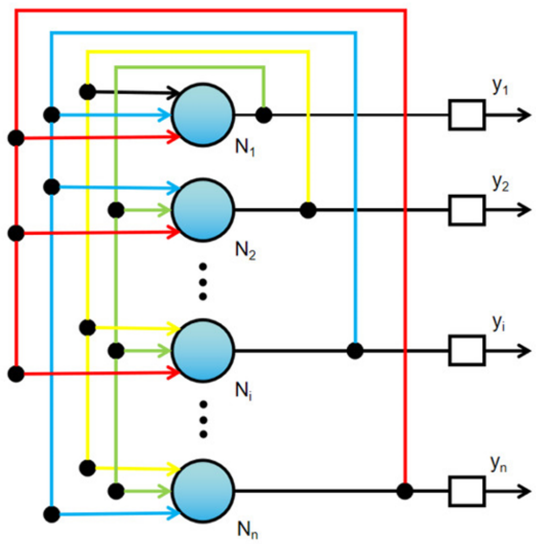

Fig. 2. Discrete Hopfield Neural Network structure (source: Yu *et al.* [6])

A core feature of DHNN is Content Addressable Memory (CAM), where synaptic weights serve as the primary unit. CAM enables the network to access memory locations directly and handle continuous raw inputs to derive a single best induced logic expression. This logic expression effectively reflects both positive and negative outcomes from a dataset. However, one known limitation of DHNN is its reliance on symbolic rule representation which requires proper input formatting and relevant information encoding before processing.

To address this challenge, the implementation of Satisfiability (SAT) theory into DHNN has significantly enhanced the understanding and operation of the net work's internal mechanisms. SAT provides a structure way to represent knowledge symbolically and observe how the network behaves under different logical constraints. Recent studies have introduced new symbolic rules in the form of

Random 3Satisfiability (RAN3SAT) by Karim *et al.,* [7] and Random 2 Satisfiability (RAN2SAT) by Sathasivam *at el.,* [8] each offering varied performance across real-world datasets.

Logic mining is a fundamental aspect of ANN models particularly within DHNN, focuses on extracting optimal logical patterns from datasets. These patterns, known as best induced logic that highlight the attributes contributing most significantly to a given outcome. Logic mining has broad applicability across various domains, offering valuable insights for both prediction and decision making.

In this study, the proposed framework combines logic reasoning with neural computation to extract interpretable rules from behavioural data. The model applies a similarity-based selection process to identify related attributes before logic formation, ensuring more meaningful and human-understandable results. Detailed descriptions of the model architecture, algorithmic steps, and preprocessing procedures are presented in the Methodology section.

The core structure of logic mining typically comprises three components which is a symbolic SAT rule set, a computational ANN model such as DHNN, and the Reverse Analysis (RA) method. This integrated system provides a robust framework for learning from data and producing interpretable logic that enhances understanding and supports informed decisions. An improved logic mining model must have the ability to handle continuous raw entries to acquire single best logic.

However, despite the advancements in logic mining and DHNN-based models, there remains a key research gap. Existing models such as 2SATRA and S2SATRA still rely on random attribute selection during clause formation, which may produce redundant or unclear logic rules, especially when applied to behavioural datasets like divorce prediction. These models also lack a mechanism to identify similarity between data instances during preprocessing, causing the model to treat dissimilar and similar data equally. This reduces the clarity and interpretability of the final induced logic. Therefore, there is a need for an enhanced logic mining model that integrates similarity-based attribute grouping using the Simple Matching Coefficient (SMC) to guide more meaningful rule construction. By addressing these limitations, the study aims to produce more interpretable, accurate, and behaviourally meaningful logical rules that can enhance understanding of marital dynamics and support explainable decision-making in social and counselling contexts.

The significance of this study lies in its ability to produce interpretable, human-understandable logic rules rather than black-box predictions. By applying the enhanced logic mining approach to the Divorce Prediction Dataset, this study not only improves classification accuracy but also provides counsellors, policymakers, and social researchers with clear explanations of why a marriage is likely to remain stable or end in divorce. Such transparency is essential in sensitive social issues, where decisions must be supported by logical and explainable reasoning. In line with the identified research gap, this study aims to develop an enhanced logic mining model that improves both accuracy and interpretability in classifying behavioural data. The main objectives of this research are as follows:

a) To apply a variant of unsupervised logic mining model namely K2SATRA in doing classification of real-life dataset with respect to the class of divorce predictor dataset.
b) To propose a new attribute selection during the pre-processing phase of the logic mining model using simple matching coefficient aiming to select N number of optimal valued similarity measures to be trained in the training phase.
c) To investigate the relationship of attributes in Divorce dataset that con tributing to divorce based on the retrieved final induced logic by the applied logic mining model.

An effective unsupervised 2SATRA model will be compare with the existing 2SATRA model that using supervised technique using Divorce Dataset. In section 2, simulation of HNN and satisfiability programming will be described in details. An Unsupervised Technique will be discussed in section 3. Section 4 is the description of experimental setup and performance metric. Result and discussion follow in section 5. The concluding remarks in section 6 complete the paper.

2. Methodology

2.1 Background Study

This section presents the theoretical background for each component involved in the proposed logic mining approach. It begins with a general overview of 2SAT based on previous research. Following that, it provides a detailed explanation of the DHNN, including a review of existing DHNN-2SAT models. Finally, the section explores the concepts behind each core component and outlines the four-phase structure of the logic mining model implemented within the DHNN framework.

2.2 Satisfiability Logic in Discrete Hopfield Neural Network

The 2SAT logic formulation is a constraint satisfaction problem involving clauses with exactly two literals. A literal is defined as a Boolean variable x_i or its negation $\neg x_i$. A clause in 2SAT is satisfied if at least one of its literals is true. Each clause represents a logical relationship between two binary attributes selected from the dataset.

In the logic mining process, the dataset is first converted into binary form $D = \{X_1, X_2, \dots, X_n\}$. Each data instance is represented as a binary vector of attributes, $X_n = (x_1, x_2, \dots, x_m) \in \{0,1\}^m$. Previous research by Kho *et al.,* [8] state that the goal is to construct logical clauses of the form $p \vee q$, where $p, q \in \{x_i, \neg x_i\}$.

The core satisfiability hypothesis in Eq. (1) is expressed as a conjunction of 2 literal clauses, where each clause shows in Eq. (2).

$$H = \wedge_{k=1}^{K} C_k, where\ K = 3 \tag{1}$$

$$C_k = \vee^{J} (p_k, q_k), \tag{2}$$

Kasihmuddin *et al.,* [10] mention in their research that this hypothesis is satisfiable if there exists an assignment of the truth values such that every clause in H is true. These clauses are then used to form the logical hypothesis that will be learned by the model. The DHNN is a recurrent neural network model characterized by binary-valued neurons and symmetric synaptic connections. In this research, DHNN is employed as a memory system to store satisfiability-based logic clauses and perform logical inference via an energy minimization process like research that conduct by Hopfield and Tank [4].

Each neuron s_i in the DHNN represents a literal from the binary attribute space and takes on a bipolar state from the set $\{-1, +1\}$, where $+1$ represents logical TRUE and -1 represents logical FALSE. The network operates based on the dynamics of energy minimization, where synaptic weights w_{ij} are symmetric $\left(w_{ij} = w_{ji}\right)$ and no self-connections exist $(w_{ii} = 0)$.

2.2.1 Energy function

The DHNN minimizes an energy function E, which is defined as Eq. (3). The energy function guarantees that as the network updates, the energy E decreases until a stable state is reached. This stable state represents a local minimum in the energy landscape, corresponding to a logic rule consistent with the input pattern.

$$E = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\, s_i s_j + \sum_{i=1}^{n}\theta_i\, s_i, \tag{3}$$

where:

- s_i is the current state of neuron i,
- w_{ij} is the synaptic weight between neurons i and j,
- θ_i is the threshold for neuron i ,
- n is the total number of neurons in the network [11].
-

2.2.2 Cost function

The cost function is derived from the number of unsatisfied clauses in the satisfiability hypothesis. Suppose the logic hypothesis H is composed of k clauses, where each clause C_k is of the form $(p_k \vee q_k)$, with literals $p_k, q_k \in \{x, \neg x\}$. The cost function is defined as Eq. (4), where NC is the total number of clauses. The definition if clause M_{ij} is given as follow Eq. (5). The goal is to minimize this cost by learning weights w_{ij} that penalize unsatisfied clauses and reward satisfied ones.

$$COST = \sum_{i=1}^{NC}\prod_{j=1}^{2} M_{ij}, \tag{4}$$

$$M_{ij} = \begin{cases}\frac{1}{2}(1 - S_{y)}, & if \neg y,\\ \frac{1}{2}(1 + S_{y)}, & otherwise.\end{cases} \tag{5}$$

2.2.3 Neuron update

Neuron states are updated using the asynchronous rule in Eq. (6). Clause logic is embedded into the DHNN weight matrix by constructing synaptic weights. Clauses such as $(x_i \vee \neg x_j)$ are encoded so that violations raise energy. The weight matrix is constructed based on SMC guided attribute pairings, reducing clause noise and improving logic integrity.

$$S_i(t+1) = \begin{cases}+1, & if\ \sum_{j=1}^{n} w_{ij} s_{j(t) > \theta_i},\\ -1, & otherwise.\end{cases} \tag{6}$$

Hyperbolic Tangent Activation Function (HTAF) is employed to ensure smoother and continuous state transitions during training. HTAF is defined in Eq. (7). This function squashes neuron inputs into the range $(-1,1)$, helping the network converge more gradually toward stable energy states and reducing oscillations during updates. The HTAF enhances the learning stability of the DHNN while still preserving its binary decision behavior after binary decision mapping.

$$f(x) = \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}. \tag{7}$$

2.3. Data Information

The Divorce Predictor dataset sourced from UC Irvine (Divorce Predictors data set - UCI Machine Learning Repository) offers a comprehensive tool for exploring marital outcomes through data-driven analysis. Derived from the Divorce Predictors Scale (DPS) based on Gottman's couples therapy framework, the dataset includes detailed questionnaire responses from 170 individuals with 84 classified as divorced and 86 as still married. Each respondent completed 54 questions with labelled Atr1 to Atr54 corresponding to Question 1 to Question 54. The rating statements for Atr1 until Atr54 on a five-point scale ranging from 0 (never) to 4 (always). The last column is the status column, which indicates whether the individual is 'Married' or 'Divorced'. It is represented by a Boolean variable, where 'Married' is represented as '1' and 'Divorced' as '0'. In terms of data normalization, k categorical clustering will be used to normalize the continuous datasets into 1 and-1 as mentioned by Sejnowski *et al.* [12]. The detail of each attribute shown as Table 1 and k-categorical clustering shown as Table 2.

Table 1
DPS attributes with detail

Attributes	Detail of Attributes
Atr1	When one of our apologies apologizes when our discussions go in a bad direction, the issue does not extend.
Atr2	I know we can ignore our differences, even if things get hard sometimes.
Atr3	When we need it, we can take our discussions with my wife from the beginning and correct it.
Atr4	When I argue with my wife, it will eventually work for me to contact him.
Atr5	The time I spent with my wife is special for us.
Atr6	We don't have time at home as partners.
Atr7	We are like two strangers who share the same environment at home rather than family.
Atr8	I enjoy our holidays with my wife.
Atr9	I enjoy traveling with my wife.
Atr10	My wife and most of our goals are common.
Atr11	I think that one day in the future, when I look back, I see that my wife and I are in harmony with each other.
Atr12	My wife and I have similar values in terms of personal freedom.
Atr13	My husband and I have similar entertainment.
Atr14	Most of our goals for people (children, friends, etc.) are the same.
Atr15	Our dreams of living with my wife are similar and harmonious
Atr16	We're compatible with my wife about what love should be
Atr17	We share the same views with my wife about being happy in your life
Atr18	My wife and I have similar ideas about how marriage should be
Atr19	My wife and I have similar ideas about how roles should be in marriage
Atr20	My wife and I have similar values in trust
Atr21	I know exactly what my wife likes.
Atr22	I know how my wife wants to be taken care of when she's sick.
Atr23	I know my wife's favourite food.
Atr24	I can tell you what kind of stress my wife is facing in her life.
Atr25	I have knowledge of my wife's inner world.
Atr26	I know my wife's basic concerns.
Atr27	I know what my wife's current sources of stress are
Atr28	I know my wife's hopes and wishes.
Atr29	I know my wife very well.
Atr30	I know my wife's friends and their social relationships.
Atr31	I feel aggressive when I argue with my wife.

Atr32	When discussing with my wife, I usually use expressions such as I Love You always
Atr33	I can use negative statements about my wife's personality during our discussions.
Atr34	I can use offensive expressions during our discussions.
Atr35	I can insult our discussions.
Atr36	I can be humiliating when we argue.
Atr37	My argument with my wife is not calm.
Atr38	I hate my wife's way of bringing it up.
Atr39	Fights often occur suddenly.
Atr40	We're just starting a fight before I know what's going on.
Atr41	When I talk to my wife about something, my calm suddenly breaks.
Atr42	When I argue with my wife, it only snaps in and I don't say a word.
Atr43	I'm mostly thirsty to calm the environment a little bit.
Atr44	Sometimes I think it's good for me to leave home for a while.
Atr45	I'd rather stay silent than argue with my wife.
Atr46	Even if I'm right in the argument, I'm thirsty not to upset the other side.
Atr47	When I argue with my wife, I remain silent because I am afraid of not being able to control my anger.
Atr48	I feel right in our discussions.
Atr49	I have nothing to do with what I've been accused of.
Atr50	I'm not actually the one who's guilty about what I'm accused of.
Atr51	I'm not the one who's wrong about problems at home.
Atr52	I wouldn't hesitate to tell her about my wife's inadequacy.
Atr53	When I discuss it, I remind her of my wife's inadequate issues.
Atr54	I'm not afraid to tell her about my wife's incompetence.

Table 2
Attributes of DPS

	Bipolar Classs	Description
Atr1 - Atr54	$K_{cluster}(Atr_i) = \begin{cases} 1, & if\ \{0,1,2\} \\ -1, & otherwise \end{cases}$	Question based on DPS
Status (class)	$K_{cluster}(Class) = \begin{cases} 1, & married \\ -1, & otherwise \end{cases}$	Classification of participation based on marital status

3. Proposed Method

2SATRA is a logic mining method designed to extract logical rules from a dataset. In the conventional 2SATRA method proposed by Kho et al., [13] variables are selected randomly. This random selection often leads to poor quality induced logic because the neurons are arranged without any specific order before the HNN begins learning. To improve this, the S2SATRA model introduces by Kasihmuddin *et al.,* [10] using chi-square analysis during the pre-processing phase to guide the selection process. This method helps identify the two most relevant neurons or clauses that are associated with the desired output H. It will allow them to participate more effectively in the HNN-2SAT learning phase which resulting in better logical rule induction.

Alternatively, the enhanced 2 satisfiability reverse analysis, K2SATRA model replaces the supervised technique which is chi-square to with unsupervised method like similarity index-based selection method. K2SATRA introduces a pre-processing layer based on the SMC to guide the construction of attribute pairs before training the HNN. This SMC based approach identifies pairs of data instances with high structural similarity. This method aims to achieve the same objective which is identifying the most relevant neuron pairs, but does evaluating the similarity between attributes and the target output. The K2SATRA approach serves as an enhanced optimization layer that reduces the need for random selection in the pretraining phase and producing higher quality logic rules with greater consistency. Figure 3 show the components that involved in K2SATRA model.

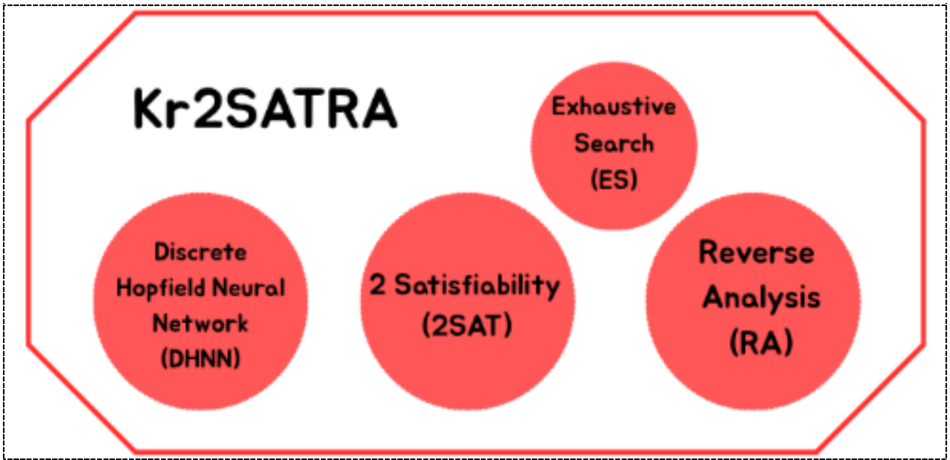

Fig. 3. Components involved in K2SATRA model

Let $S_i = (S_1, S_2, S_3, \dots, S_N)$ be the set of neurons representing N attributes in the dataset. Each neuron is initially represented in binary and later converted to bipolar form, $S_i = \{-1,1\}$. K2SATRA aims to select the best pair of neurons to construct the clause $C_i^{(2)}$, guided by the similarity patterns in the dataset. To measure similarity between instances, we use the SMC formula as Eq. (8):

$$SMC = \frac{(a+d)}{(a+b+c+d)}, \tag{8}$$

Where:

- a = number of attributes where both instances are 1
- d = number of attributes where both instances are 0
- b, c = mismatched attributes [14]

From the SMC similarity matrix, K2SATRA forms clusters of behaviorally similar instances. Attributes that consistently appear across similar instances are then selected to construct candidate clauses. These clauses are tested and filtered before being used to form the logical hypothesis Q_{l_i}. After selecting the best attributes based on shared instance patterns, neuron states are converted to bipolar representation in Eq. (9).

$$S_i = \begin{cases} +1, & if\ x_i = 1 \\ -1, & otherwise. \end{cases} \tag{9}$$

Only second-order clauses $C_i^{(2)}$ are considered in the K2SATRA logic structure. For each clause that leads to a correct classification $Q_{l_i} = 1$, we record its frequency. The most frequently occurring valid clause is defined as the optimal logic hypothesis using Eq. (10). Once the optimal logic rules are identified, they are encoded into the DHNN through synaptic weights. Eq. (11) is the final state of the neurons after training and it's denoted by S_{B_i}.

$$Q_{best} = \max(n|C_i^{(2)} such\ that\ Q_{l_i} = 1) \tag{10}$$

$$S_i^{induced} = \begin{cases} p_k, q_k, & if\ S_{B_i} = +1, \\ \neg p_k, \neg q_k, & if\ S_{B_i} = -1. \end{cases} \tag{11}$$

To ensure that only globally optimal states are retained, K2SATRA applies a verification step using a cost function adapted from the energy equation as Eq. (12), where $H(Q_{B_i})$ is the energy of the current clause hypothesis, and Tol is a tolerance threshold for selection. If no strong similarity is found between any attribute pairs, Kr2SATRA defaults to the baseline 2SATRA framework as described by several authors [15, 16].

$$Q_{B_i} = \begin{cases} Q_{B_i}, & if\ |H(Q_{B_i}) - H_{min}(Q_{B_i})| < Tol, \\ 0, & otherwise. \end{cases} \tag{12}$$

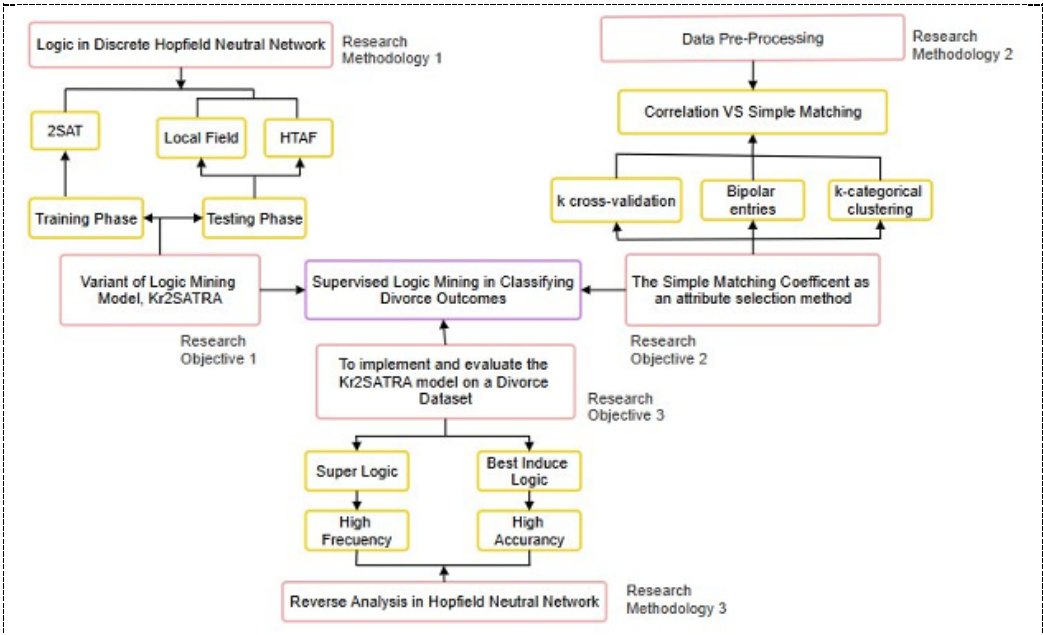

Fig. 4. The overview of methodology for K2SATRA model

4. Experiment and Discussion

4.1 Performance Metrics

To evaluate the robustness and classification capability of the K2SATRA model, several comprehensive performance metrics were adopted. These include accuracy, sensitivity, specificity, false positive rate (FPR), and Bookmaker Informed ness (BM). These metrics allow for a multifaceted analysis of model behavior, especially important in datasets with imbalanced or uncertain distributions. To evaluate the model's performance, the key evaluation metric is used include *True Positives* (TP), *True Negatives* (TN), *False Positives* (FP), and *False Negatives* (FN).

Accuracy (Acc) reflects the overall proportion of correctly classified instances. It provides a general measurement of correctness across both positive and negative classes and is useful as an initial indicator of performance. The formula for accuracy shown as Eq. (13).

$$Accuracy\ (Acc)\ =\ \frac{TP + TN}{TP + TN + FP + FN} \tag{13}$$

Sensitivity (Se) also known as *recall*, based on Jha *et al.,* [17] it measures the model's ability to correctly identify actual positive instances calculated from Eq. (14). It is especially important in applications were missing a positive case (e.g. = potential divorce indicators) can have significant consequences:

$$Sensitivity\ (Se) = \frac{TP}{TP+FN} \tag{14}$$

Next, **Specificity (Sp)** measures how well the model identifies negative instances correctly. It complements sensitivity by ensuring that the model does not falsely label many non-critical cases as positive. Eq. (15) prove the specificity focuses on *true negative* as it complements the sensitivity formula which focuses on *true negative*.

$$Specificity\ (Sp) = \frac{TN}{TN + FP} \tag{15}$$

False Positive Rate (FPR) in Eq. (16) indicates the proportion of negative cases that were incorrectly predicted as positive. A low FPR is essential in reducing false alarms and ensuring that predictions are reliable:

$$False\ Positive\ Rate\ (FPR)\ =\ \frac{FP}{FP + TN} \tag{16}$$

Bookmaker Informedness (BM), also known as *Youden's J statistic,* combines both sensitivity and specificity into a single metric to measure how informed or skilled the model's predictions are as shown in Eq. (17). It is particularly valuable in assessing model quality when classes are imbalanced:

$$Bookmaker\ Informedness\ (BM) = Sensitivity\ +\ Specificity\ -\ 1 \tag{17}$$

The BM score ranges from –1 (completely uninformed or misleading) to 1 (perfect classification), with 0 representing no better performance than random guessing. These metrics were selected to ensure a holistic assessment of the K2SATRA model's performance in discovering behavioral logic patterns from the Divorce Dataset. Although K2SATRAoperates under an unsupervised framework, classification-based evaluation is still meaningful when comparing post-processing results to available ground truth labels.

4.2 Experimental Setup

For comparative analysis, the K2SATRA model will be benchmarked against the S2SATRA model proposed by Kasihmuddin *et al.,* [10] which integrates correlation filtering into the clause selection phase. Both models utilize supervised learning and the 2SAT framework embedded into a DHNN. Additionally, both employ permutation operators to explore various attribute combinations. However, they differ significantly in the pre-processing strategy used for clause construction. Kasihmuddin *et al.,* [10] mentioned that the S2SATRA model selects clause pairs based on correlation strength between attributes. These correlations are ranked and filtered before clause formulation. During retrieval, S2SATRA applies the supervised 2SAT rules to the DHNN and uses the Lyapunov energy function that conducted by Abdullah [18] to ensure convergence to a valid solution. While this correlation-based strategy improves upon random pairing, it does not consider similarity at the instance level, which can lead to logic rules that are statistically valid but behaviourally inconsistent.

In contrast, K2SATRA replaces the correlation filter with a SMC based similarity index to applied during the preprocessing phase. The SMC evaluates pairwise instance similarity across binary attributes and clause formation is guided by pat terns that are common within behaviourally similar groups. This allows K2SATRA to better preserve interpretability and pattern consistency. To ensure fairness, both models use the same clause structure as Eq. (1). It will operate on the same dataset which is Divorce Predictor Dataset and apply identical learning and retrieval rules within the DHNN framework. All experiments maintain fixed learning thresholds, neuron update rules and activation functions, allowing us to isolate the effect of the similarity driven pre-processing introduced in K2SATRA. The parameters that have been used in this research shown as Table 3 for K2SATRA model and Table 4 for S2SATRA model by Kasihmuddin *et al.* [10]. The evaluation focuses on performance metrics such as accuracy, sensitivity, specificity, FPR and BM as well as the clarity and generalization of the logic rules produced.

Table 3
List of parameters in K2SATRA model

Parameter	Parameter Value
Neuron Combination	100
Number of Trial	100
Attributes Selection	SMC
Number of Learning Ω	100
P-Value P	0.05
Logical Rule	H
Tolerance Value ϱ	0.01
No Neuron String	100
Maximum Permutation Per	100

Table 4
List of parameters in S2SATRA model by Kasihmuddin *et al.* [10]

Parameter	Parameter Value
Neuron Combination	100
Number of Trial	100
Attributes Selection	$Correlation$
Number of Learning Ω	100
P-Value P	0.05
Logical Rule	H
Tolerance Value ϱ	0.01
No Neuron String	100
Maximum Permutation Per	100

All simulations for the K2SATRA model were implemented using Dev C++ Version 5.11 Blue icon (developed by Bloodshed Software, USA). For cross-validation procedures, Dev C++ 6.3 (Red icon) was also utilized to ensure consistency in coding and execution environments. To handle data management and storage, both IBM SPSS Statistics Version 27 (IBM Corporation, New York, NY, USA) and Microsoft Excel 2016 (Microsoft Corporation, USA) were used. These tools facilitated the recording of experimental results, particularly for tabulating error values, model outputs, and performance metrics.

All experiments were executed on the same computing device to eliminate environmental inconsistencies that could introduce variations in model performance. To minimize the impact of random initialization in the logic mining process, each K2SATRA model underwent 5 runs by cross-validation process and the results were averaged to ensure the reliability and robustness of the findings.

5. Result and Discussion

5.1 Simple Matching Coefficient Results

The similarity index test using the SMC, will be used to identify the most relevant attributes for this study, rather than selecting them randomly or through statis tical method such as correlation analysis test. Based on the similarity scores, the top 6 attributes with the highest similarity to the output class will be selected. Higher SMC values are preferred, as they indicate stronger similarity between input attributes and the dataset's target class. Selecting the most relevant attributes through this similarity-based method ensures that the synaptic weights generated during DHNN training are more valid and accurate, ultimately leading to more optimal induced logic during the retrieval phase such as research conduct by Rusdi *et al.* [19]. The overall value of SMC is represented in Figure 5 and Table 5 is the following the selected top 6 attributes after performing similarity analysis using the SMC method.

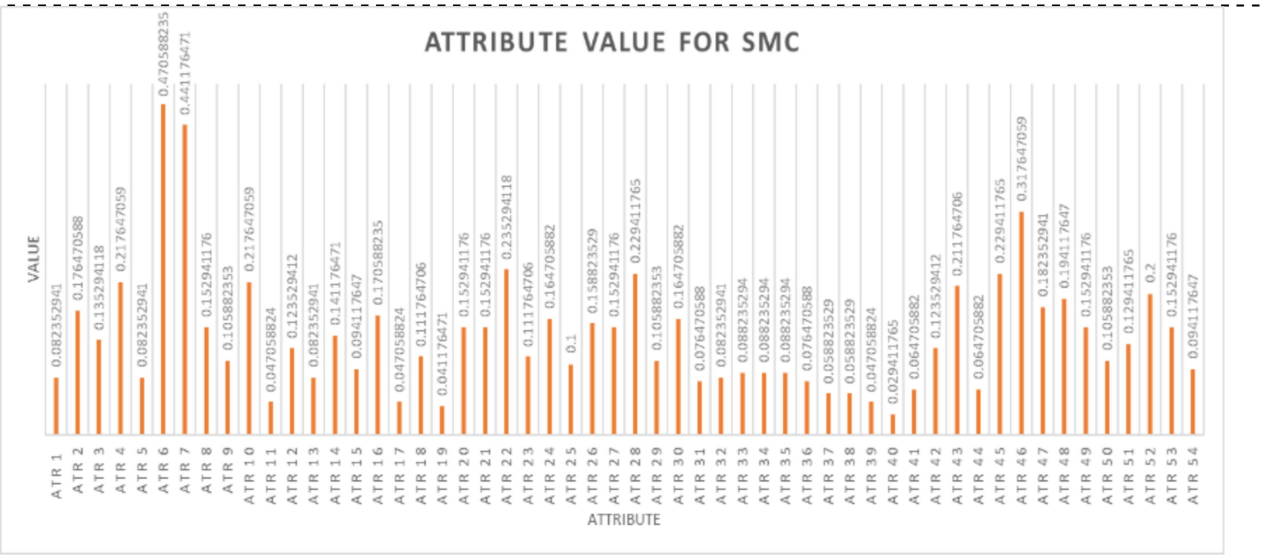

Fig. 5. Bar Chart of attribute values for SMC

Table 5
Top 6 of SMC results

Attributes	SMC Values
Atr6	0.470588
Atr7	0.441176
Atr46	0.317647
Atr22	0.235294
Atr28	0.229412
Atr45	0.229412

After selecting the top six attributes, the binary dataset is subjected to cross validation to evaluate the model's performance. The dataset is divided into different training and testing ratios:

60% training and 40% testing, 70% training and 30% testing, 80% training and 20% testing, and 90% training and 10% testing, with each configuration evaluated using 5-fold cross-validation. This approach ensures that the model is tested on multiple subsets of data, which helps reduce bias and variance, especially in smaller datasets. K-fold cross-validation is chosen because it provides a more robust and reliable evaluation by averaging the performance across different data splits. This method helps avoid over fitting and gives a clearer picture of the model's generalization ability across different proportions of training and testing data.

5.2 Discussion of Performance Metrics

To captures the outcomes of a classifier, terms of *True Positives* (TP), *True Negatives* (TN), *False Positives* (FP), and *False Negatives* (FN) used. The outcomes of the evaluation are summarized using five key performance metrics which is Acc, Se, Sp, FPR and BM as detailed in Table 6 and Table 7.

The K2SATRA model demonstrates consistently high sensitivity across all dataset partitions, indicating its strong ability to correctly identify divorce cases. However, specificity values remain comparatively low, suggesting difficulty in detecting non-divorce (married) cases. This imbalance also leads to elevated false positive rates, particularly in the 6040 and 7030 datasets.

As the training proportion increases (up to 90%), accuracy and balanced metric values improve, which implies that K2SATRA benefits from larger training sets. Notably, the 9010 partition shows the best trade-off between sensitivity and specificity with the highest BM score (0.4412). Nonetheless, the model tends to favour the positive class, indicating a classification bias.

Table 6
The average of performance Metric by K2SATRA

Dataset	ACC ↑	SE ↑	SP ↑	FPR ↓	BM ↑
6040	0.661764706	1.000000000	0.280303030	0.719696970	0.280303030
7030	0.671568627	0.911111111	0.274509804	0.725490196	0.185620915
8020	0.705882353	1.000000000	0.292852625	0.707147375	0.292852625
9010	0.720588235	1.000000000	0.441176471	0.558823529	0.441176471

Table 7
The average of performance Metric by S2SATRA

Dataset	ACC ↑	SE ↑	SP ↑	FPR ↓	BM ↑
6040	0.492647059	1.000000000	0.007352941	0.992647059	0.000000000
7030	0.519607843	1.000000000	0.013071895	0.986928105	0.000000000
8020	0.507352941	1.000000000	0.019607843	0.980392157	0.000000000
9010	0.514705882	1.000000000	0.029411765	0.970588235	0.000000000

The comparative results shown in Table 7 indicate that the proposed K2SATRA model outperforms the S2SATRA framework in terms of average accuracy, specificity and False Positive Rate. The consistent performance across training and testing sets suggests that the enhanced model achieves better generalization, meaning it can classify new, unseen data more reliably. This improvement can be attributed to the inclusion of similarity-based attribute selection using the Simple Matching Coefficient, which reduces redundancy and enhances the model's ability to detect relevant behavioural patterns.

The obtained accuracy of 66-72%, along with balanced sensitivity and specificity values, demonstrates that the proposed K2SATRA model performs competitively compared to earlier logic-based frameworks, S2SATRA, which achieved around 49-51% accuracy in similar datasets. Although the performance margin appears modest, the improvement is meaningful because it is achieved without compromising interpretability. This indicates that the K2SATRA framework effectively balances predictive accuracy with logical transparency, providing results that are not only data-driven but also human-understandable. In practical terms, an accuracy of 72% suggests that the model can correctly identify relationship outcomes in roughly two out of three cases, which is a promising result for a logic-based interpretive system applied to complex human behavioral data.

5.3 Discussion Best Induced Logic

The logic generated by the enhanced model named K2SATRA framework shows a clear and easy-to-understand decision-making process. This rule is selected using unsupervised technique which is SMC feature selection. It also matches common practices used in social or psychological like marital cases, which supports the reliability of the model. Because the rule is simple, it improves transparency and allows counsellors to easily understand and explain marital outcomes or decisions. At the same time, it still maintains strong predictive accuracy in identifying relationship stability or risk of divorce.

Best Induced Logic:
$(\neg F \vee C) \wedge (\neg D \vee A) \wedge (\neg B \vee E)$
from train:test split (60:40) Fold 2

A – *atr6 : We don't have time at home as partners.*
B – *atr7 : We are like two strangers who share the same environment at home rather than family.*
C – *atr46 : Even if I'm right in the argument, I'm careful not to upset the other side.*
D – *atr22 : I know how my wife wants to be taken care of when she's sick.*
E – *atr28 : I know my wife's hopes and wishes.*
F – *atr45 : I'd rather stay silent than argue with my wife.*

This logical rule describes the behavioural conditions that influence whether a relationship remains stable or becomes at risk of divorce. Each clause represents a distinct behavioural pattern that contributes to marital outcomes. The interpretations are discussed as follows.

1. **Clause 1:** $(\neg \boldsymbol{F} \vee \boldsymbol{C})$

This clause indicates that a relationship is more likely to remain stable when either one of two conditions is met:

- the partner does not remain silent during conflicts $(\neg F)$, or
- the partner chooses to avoid hurting their spouse even when they are right in an argument (C).

In this context, F (Atr45) reflects emotional withdrawal, where a partner prefers silence rather than addressing issues. Silence may prevent immediate conflict, but prolonged avoidance can create emotional distance. Meanwhile, C (Atr46) reflects emotional sensitivity and empathy in arguments. Even during disagreements, placing importance on the partner’s feelings helps protect the relationship from escalation.

This clause suggests that healthy conflict resolution is not defined by the absence of arguments, but by the presence of empathy and willingness to communicate. Therefore, marriages remain more stable when partners either engage in open communication or manage disagreements with emotional care.

2. **Clause 2:** $(\neg \boldsymbol{D} \vee \boldsymbol{A})$

This clause explains that relationship strain may occur when partners do not know how to take care of each other during illness (¬D), especially when combined with a lack of time spent together as partners (A). However, when a partner is attentive and understands how to provide care during illness (D), the relationship is more likely to be secure.

(A) represents the absence of shared quality time at home, which reduces emotional closeness. When this is combined with ¬D, a lack of caregiving knowledge, it indicates both emotional and practical distance in the relationship.

Thus, this clause highlights that caring for a partner during vulnerable moments, such as illness, is an important indicator of emotional intimacy. The ability to provide such care compensates for other shortcomings and supports marital stability.

3. **Clause 3:** $(\neg \boldsymbol{B} \vee \boldsymbol{E})$

This clause shows that the relationship is more stable when partners do not feel like strangers living in the same house (¬B). However, even if they do feel emotionally disconnected (B), stability can still be maintained if they understand each other's hopes and wishes (E).

Feeling like "two strangers in the same environment" indicates emotional detachment and lack of companionship. Yet, the presence of E suggests that meaningful emotional understanding still exists between partners. Knowing and valuing the partner's dreams, expectations, and life goals compensates for physical or emotional distance.

This clause reveals that emotional understanding is a protective factor against marital breakdown. Even when daily interactions are weak, couples who maintain deep knowledge of each other's feelings and aspirations can preserve relationship stability.

The induced logic highlights that divorce is not simply caused by conflict, but more often by emotional distance, silence, lack of care, and reduced mutual understanding. Conversely, empathy, caregiving, and emotional knowledge of one's partner act as strong protective elements within a marriage. This makes the model useful for:

- Counsellors to identify key warning signs in couples,
- Researchers to understand emotional patterns in divorce prediction,
- Couples to reflect on which behaviours strengthen or weaken their bond.

5.4 Critical Reflection and Implications

Although the K2SATRA model demonstrated improved interpretability and classification accuracy, several limitations must be considered when assessing the validity and generalizability of the findings. First, the conversion of behavioural responses into binary values (1 and –1) may oversimplify the complexity of human relationships, potentially reducing the richness of emotional or contextual variation captured in the dataset. This simplification may limit the model's ability to represent nuanced behavioural patterns that exist on a spectrum rather than as discrete categories. Second, the framework currently relies on a single type of logical formulation within a fixed satisfiability structure. While this improves clarity, it may restrict the model's flexibility in capturing diverse forms of reasoning that occur in real-world relationship dynamics. Consequently, these constraints may limit the external validity of the model when applied to other datasets or social domains. Future research should explore hybrid logic representations and multi-valued data encoding to enhance generalization and ensure that the model remains adaptable to more complex human behaviours.

The findings of this study are consistent with earlier research emphasizing the importance of interpretability and logical reasoning in behavioural prediction models. Compared to the S2SATRA framework proposed by Kasihmuddin et al., [10] the K2SATRA model provides clearer and more consistent logic expressions, confirming that integrating similarity-based grouping improves the coherence of rule induction. This aligns with Jamaludin et al., [20] who noted that random attribute selection in logic models often reduces clarity and generalization. Theoretically, this research strengthens the foundation of explainable artificial intelligence (XAI) by demonstrating how unsupervised similarity-based reasoning can enhance symbolic logic extraction from complex social data.

From a practical perspective, the results show that interpretable rules can offer meaningful insights for counsellors and social researchers by identifying key behavioural patterns associated with marital stability or risk of divorce. However, the model's application is limited to structured binary data and may require adaptation for larger, more diverse datasets. Future studies could explore hybrid models that combine logical reasoning with deep learning to achieve both transparency and scalability in behavioural prediction.

6. Conclusion and Future Work

This thesis developed an intelligent computational model using the enhanced S2SATRA framework and DHNN to better understand how logic-based models can be used for real-life prediction. The proposed variant, Unsupervised 2-Satisfiability Reverse Analysis, K2SATRA was successfully applied to classify real divorce data by identifying meaningful patterns within the dataset. The findings of this research demonstrate that the proposed K2SATRA framework effectively improves the interpretability and generalization of logic-based models in classifying behavioural data This induced logic not only explains the behaviour of the dataset but also provides clear evidence of the factors that influence marital outcomes. By emphasizing similarity-based feature grouping and clear rule formation, the model produces logical expressions that are both accurate and human-understandable. These results highlight the model's potential for application in behavioural analysis, marital counselling, and other social domains where transparency and interpretability are crucial. This research contributes to the understanding of computational intelligence in behavioural analysis.

Future studies should focus on extending the K2SATRA framework to handle more complex and multi-valued data representations, allowing the model to capture a broader range of behavioural and emotional nuances. The framework should be validated using larger and more diverse datasets, including data from different cultural, psychological, or social contexts, to improve its generalizability. Beyond marital prediction, the model can also be applied to education, healthcare, and workplace analytics, where understanding human behaviour and decision-making is equally critical. These directions would enhance the robustness, adaptability, and real-world applicability of the K2SATRA model.

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