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Mathematical Modelling of Harmful Algal Bloom with Prey-Predator Interaction

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ABSTRACT

An algal bloom is a condition by extensive algal growth in a specific area that is consider harmful when the bloom causes a damaging effect. Harmful algal blooms (HABs) events are dangerous to human, ecosystem, and marine creatures. Therefore, this study is proposed an extension of the HABs mathematical modelling to describe the HABs events. This model consists of two variables: Toxic-Producing Phytoplankton (TPP) population and zooplankton (Z) population. This research considered the prey-refuge factor to evaluate its stabilizing and destabilizing impacts on the system. Stability analysis is conducted to determine the stability condition of the model and the result is verified by applying the numerical simulations. Numerical simulation was performed across varying refuge prey levels to observe system behaviors. A model of prey-predator interaction is being studied to show the effect of HAB. The result show that the low prey refuge enhances stability and balanced species coexistence while higher refuge prey rate reduce zooplankton predation effectiveness will lead to unstable dynamic and increases HAB risk. This model provides a theoretical basis for predicting blooms occurrence and contribute to the development of early warning strategies for HAB management.

1. Introduction

Algae are photosynthetic microorganisms that are found in most habitats. Algae vary from small, single-celled forms to complex multi-cellular forms. An algal bloom is a rapid increase in the density of algae in an aquatic system. Algal blooms sometimes are natural phenomena, but their frequency, duration and intensity are increased by nutrient pollution. Algal blooms can change the colour of the water and give effect for coastal and aquatic in the water [1]. The potential for algae blooms comes from nutrient pollution, an overabundance of the essential plant nutrients nitrogen and phosphorus. When the concentrations of nitrogen and phosphorus increase in a water body, the right combination of temperature, sunlight and low flow can trigger an algal bloom.

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Harmful algal blooms (HABs) occur when colonies of algae that simple plants that live in the sea and freshwater. It is growth out of control and produce toxic or harmful effects on people, fish, shellfish, marine mammals and birds. The human illnesses caused by HABs, though rare, can be debilitating or even fatal. Harmful algal blooms (HABs) have become a worldwide environmental and human health problem [2,8,9]. Prey -predator model is a mathematical model that illustrates the interaction between two species, while one species acting as a predator of the other. In the relationship between prey-predator, phytoplankton acts as the prey, while zooplankton functions as the predator. The interaction of prey and predators with phytoplankton forming a community structure and the zooplankton community structure giving rise to feeding strategies as predators. However, the palatability of phytoplankton species to zooplankton depends on several factors such as cell size, structure, shape, chemical composition, and nutritional quality of the phytoplankton [3,10]. This indicates that zooplankton primarily consume phytoplankton to establish the foundation of numerous aquatic food webs.

Then, Choi *et al.*, [4] introduced the modelling analytical population effect of predation avoidance for triggered harmful algal bloom. The model that consists in this research was phytoplankton and zooplankton. In this research, two phytoplankton group consists which are one non-harmful and one HAB for complete in the same system equilibrium conditions cannot occur without presence of zooplankton. There are only non-harmful phytoplankton group which defeats the HABs group in the resources can survive in the equilibrium. Subsequently, Li *et al.*, [5] introduced the modelling of phytoplankton-zooplankton interaction with toxin effect and refuge. This research to investigate the impact of toxins produced by phytoplankton and refuges provided for phytoplankton on interactions. According to this research, both phytoplankton refuge and toxins have a significant impact on the occurring of algal blooms.

Besides, Basak *et al.*, [6] presented the modelling of prey-predator interaction the effect of prey refuge and harvesting, and more recent studies have applied machine learning and multi-factor models to HAB prediction [11-13]. This research to propose a mathematical model of TPP and zooplankton species which follows the Holling-Type III response and the effect of prey-refuge on both species. The parameter prey refuge significantly influences the dynamics of the model system. Therefore, the growth rate of phytoplankton serves as a controlling factor for the dynamics of the model.

This research aims to develop growth model of algae population, analyse the stability of the model and investigate the impact of prey refuge. The current research is focused on analysis of a mathematical model describing the interaction between toxic phytoplankton and zooplankton in aquatic ecosystem. The model system consists of toxic-phytoplankton and zooplankton. The stability of the equilibrium state of the system is analysed. The techniques in stability of the equilibrium point are linearization, eigenvalue analysis, and the Routh-Hurwitz criteria. The system behaviours in scenario are investigate in absence of TPP, zooplankton or both species. The numerical simulations to observe system behaviours under different parameter conditions such as d differing levels of prey refuge.

The output of this research will give the benefits to the ecosystem of the water, human and animal. Economic losses due to HAB events can be mitigated or reduced through improved understanding of early detection and strategies [12-16]. Biological understanding can also assist in HAB mitigation efforts and inform individuals in delivering health alerts regarding the toxic algal bloom. Therefore, enhancing our knowledge of phytoplankton growth patterns is important for providing timely health advisories and safeguarding communities that depend on freshwater and coastal resources, as some of these organism release toxins that are toxic to both humans and animals.

2. Material and Methodology

2.1. Prey-Predator Model (Toxic Phytoplankton-Zooplankton Model)

Many researchers have constructed and studied the mathematical model of toxic phytoplankton-zooplankton interaction. Mathematical modelling is important in order to improve our knowledge and understanding of the occurrence of HAB in relation to plankton ecology. This research incorporates a mathematical model with different the value of parameter rate of refuge prey to describe how HAB occur.

$$\begin{aligned}\frac{dx}{dt} &= rx - \alpha(1 - \beta)xy, \\ \frac{dy}{dt} &= b(1 - \beta)xy - my\end{aligned}\tag{1}$$

where

r = Intrinsic growth rate of prey

α = Rate at which predator catches prey

β = Rate of refuge prey

b = Conversion factor denoting the number of newly born predator for each captured prey

m = Death rate of predator

- Let $x(t)$ be the toxin production phytoplankton (TPP) as a prey which are being consumed by the $y(t)$ which is zooplankton population as a predator.
- The value of parameter of rate of refuge prey is used in difference value when $\beta = 0, 0.01, 0.1, 0.4, 0.7, 0.9$ to analysis the stability of population.

In this subsection, there are following four equilibrium points:

- 1) Extinction of toxic-phytoplankton and zooplankton

$$E_1(0,0)$$

- 2) Extinction of zooplankton

$$E_2(x_2^*, 0)$$

- 3) Extinction of toxic-phytoplankton

$$E_3(0, y_3^*)$$

- 4) Two species coexistence equilibrium

$$E_4(x_4^*, y_4^*)$$

The model of prey-predator model interaction that coexistence all population is investigated. The conditions for the stability around coexistence for all population of toxic-phytoplankton and zooplankton at $E_4(x_4^*, y_4^*)$ is obtained.

The Jacobian matrix of the system around $E_4(x_4^*, y_4^*)$ can be written as:

$$J(E_4) = \begin{bmatrix} r - ay^*(1 - \beta) & -ax^*(1 - \beta) \\ by^*(1 - \beta) & -m + bx^*(1 - \beta) \end{bmatrix}$$

Eigenvalues of the above matrix is computed as follows:

$$J(\lambda - E_4) = \begin{bmatrix} \lambda - A_1 & -ax^*(1 - \beta) \\ by^*(1 - \beta) & \lambda + A_2 \end{bmatrix}$$

where

$$A_1 = -r + y^* - ay^*\beta,$$

$$A_2 = m - bx^* + b^*\beta.$$

The corresponding characteristic equation of E_4 can be written as:

$$(\lambda^2 + S_1\lambda + S_2) = 0, \quad (2)$$

where

$$S_1 = -(A_1 + A_2)$$

$$S_2 = -A_1A_2 + abx^*y^* - 2ab\beta x^*y^* + ab\beta^2 x^*y^*$$

System (1) around $E_4(x_4^*, y_4^*)$ is locally asymptotically stable if $S_1 > 0$ and $S_2 > 0$.

Proof. According to Routh-Hurwitz criterion, the condition for the root to have negative real parts are $S_1 > 0$ and $S_2 > 0$ for equation (2). Therefore, the system is locally asymptotically stable if satisfy all the condition of Routh-Hurwitz criterion.

3 Results and discussion

3.1. Prey-Predator Model (Toxic Phytoplankton-Zooplankton model)

A set of parameter values from the literature was used to substantiate the analytical results obtained through numerical simulation (see Table 1)

Table 1
Parameter values used in the numerical simulation (Toxic Phytoplankton-Zooplankton model)

Parameters	Symbol	Values
Intrinsic growth rate of prey	r	$0.1(mL. h^{-1})$
Rate at which predator catches prey	α	$0.1 (h^{-1})$
Rate of refuge prey	β	$0, 0.01, 0.1, 0.4, 0.7, 0.9(mL. h^{-1})$
Conversion factor denoting the number of newly born predator for each captured prey	b	$0.3(mL. h^{-1})$
Death rate of predator	m	$0.2(h^{-1})$

From the numerical simulations, it is found that the system is stable for $\beta = 0$ as in Figure 1, which means that no values of the rate of the refuge prey. There is no refuge prey is provided for the prey, so the predator can fully consume the prey. This result creates a strong prey-predator interaction is fully active. Figure 2 depicts the equilibrium between TPP and zooplankton population is asymptotical stability for $\beta = 0$. This can lead to a balance prey-predator interaction when both species can coexist and maintain stable population levels.

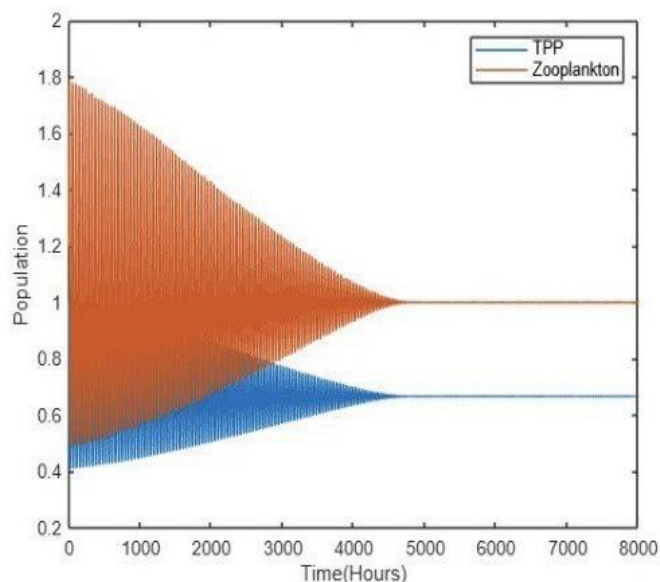


Fig. 1. Simulation results of the system (1) with $\beta = 0$

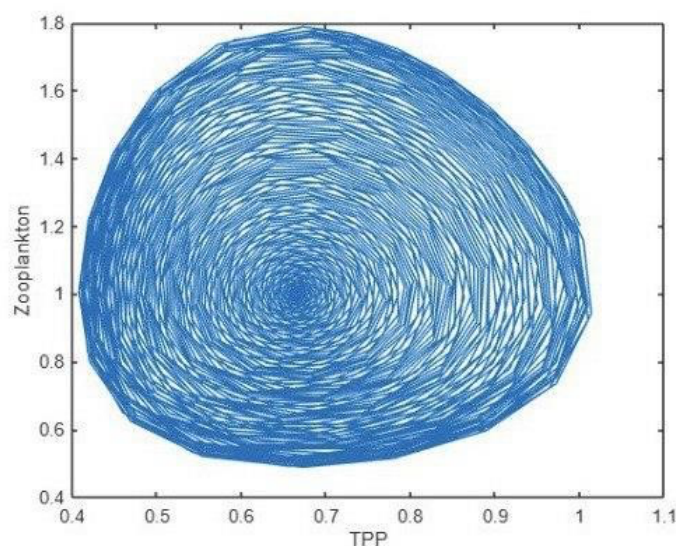


Fig. 2. The asymptotical stability of equilibrium between TPP and zooplankton population for $\beta = 0$

From the numerical simulation, the value of the rate of refuge prey parameter of the system (1) for stability behaviours same for $\beta = 0.01$. The system is stable as in Figure 3. The result for the system is same with the Figure 1. The value of rate of refuge prey parameter is low, so that most of the prey are accessible to the predator and interaction is still effective. Figure 4 depict the equilibrium between TPP and zooplankton population is stable for stability for $\beta = 0.01$. This result show that the minor refuge prey has a little impact on system stability.

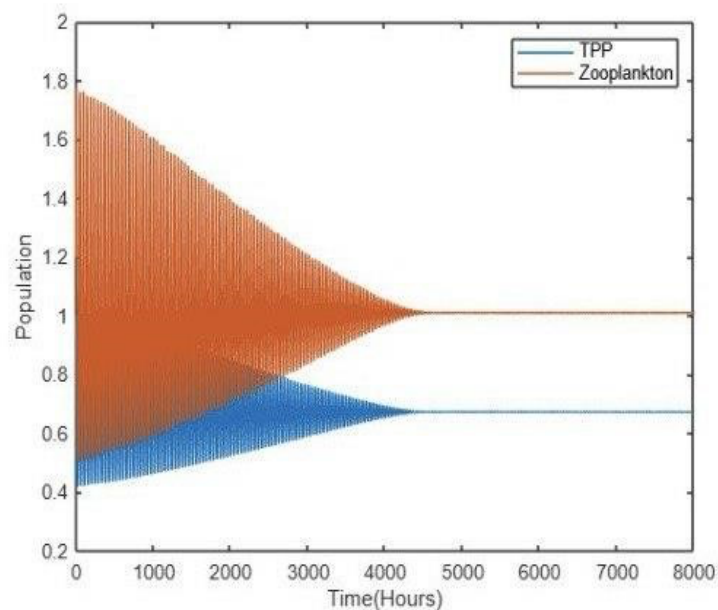


Fig. 3. Simulations results of system (1) with $\beta = 0.01$

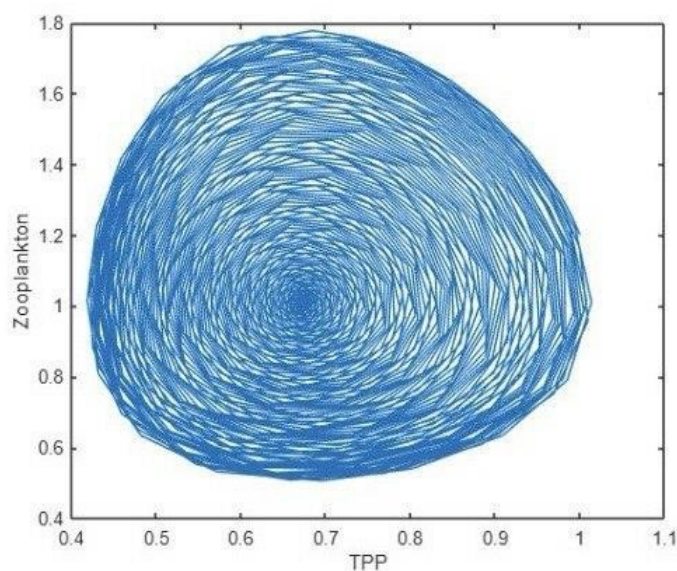


Fig. 4. The asymptotical stability of equilibrium between TPP and zooplankton population for $\beta = 0.01$

From the numerical system, it is found that the system is stable when $\beta = 0.1$ as in Figure 5. The refuge rate is increase but predator still maintain enough to access to prey. Figure 6 shows the asymptotically stability of the equilibrium between TPP and zooplankton population for $\beta = 0.1$. The zooplankton population still survive and control TPP.

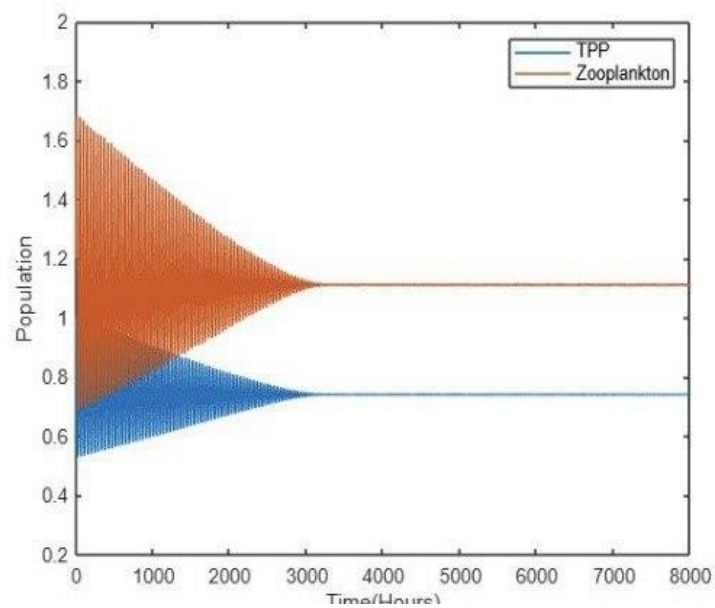


Fig. 5. Simulation results of system (1) with $\beta = 0.1$

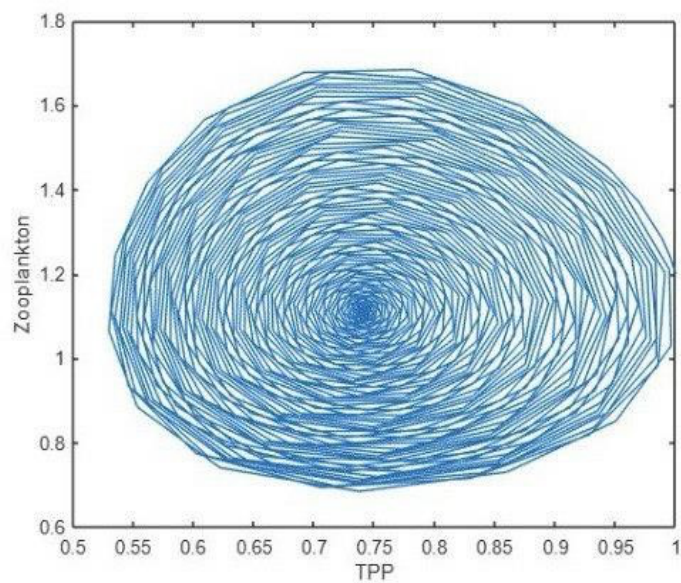


Fig. 6. The asymptotical stability of equilibrium between TPP and zooplankton population for $\beta = 0.1$

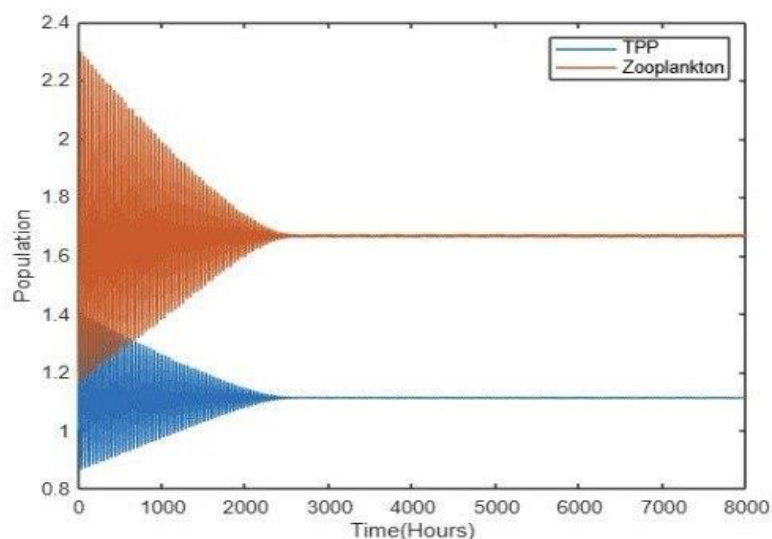


Fig. 7. Simulation results of system (1) with $\beta = 0.4$

From the numerical system, it is found that the system is stable for $\beta = 0.4$ as in Figure 7. This result show more TPP is hidden and protected, so zooplankton food source is more limited. Figure 8 shows the asymptotically stability of the equilibrium between TPP and zooplankton population for $\beta = 0.4$. The system still stabilizes but slower and with small oscillation.

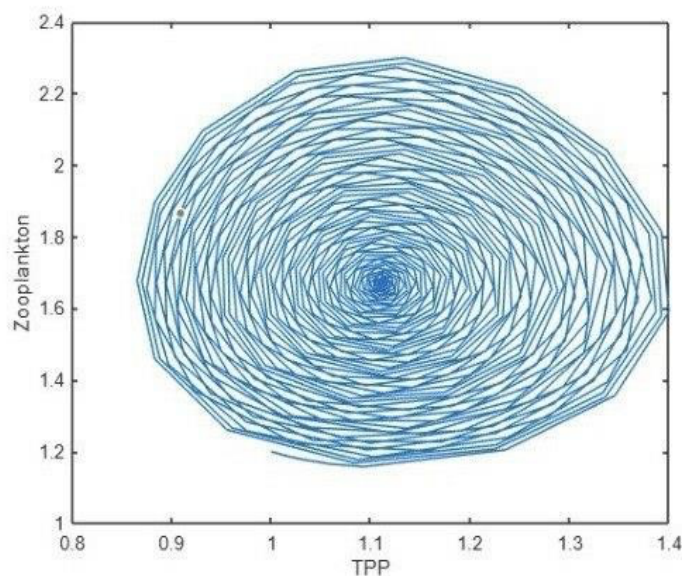


Fig. 8 The asymptotical stability of equilibrium the between TPP and zooplankton population for $\beta = 0.4$

From numerical simulations, it is found that the system is unstable for $\beta = 0.7$ as in Figure 9. Most TPP are now hidden and protected, so that zooplankton cannot control the TPP. This means that there is HAB occurrence when the rate of refuge prey is higher. The system rate of refuge factor has stabilizing and destabilizing effects. Figure 10 shows the equilibrium between TPP and zooplankton population loses their stabilities when $\beta = 0.7$. This show that a prey-predator interaction exists between TPP and zooplankton population. The prey-predator interaction is reduced predation when refuge prey is increase. The prey will overgrowth but predator will decline the interaction which

result unstable dynamic. The system starts to destabilizing influence when the refuge prey factor increases, it diminishes predation pressure on toxic-phytoplankton and allowing the population to grow uncontrollably which can lead to HAB.

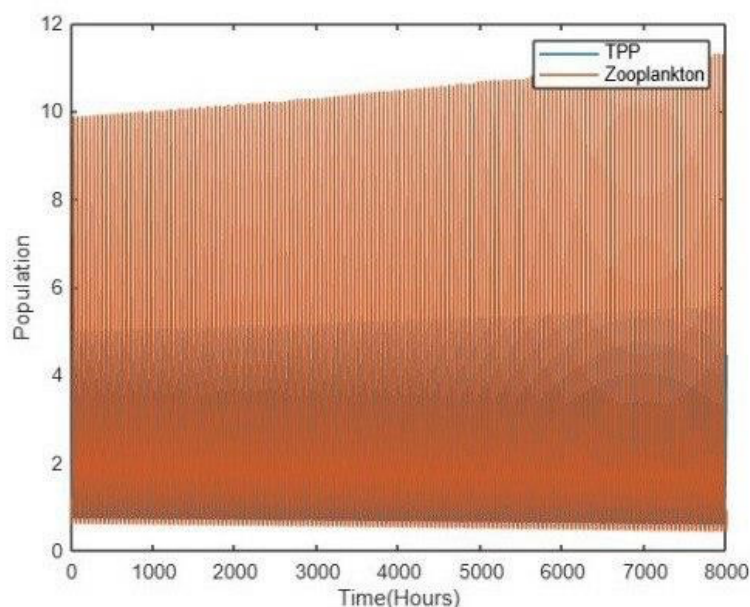


Fig. 9. Simulations results of system (4.1) with $\beta = 0.7$

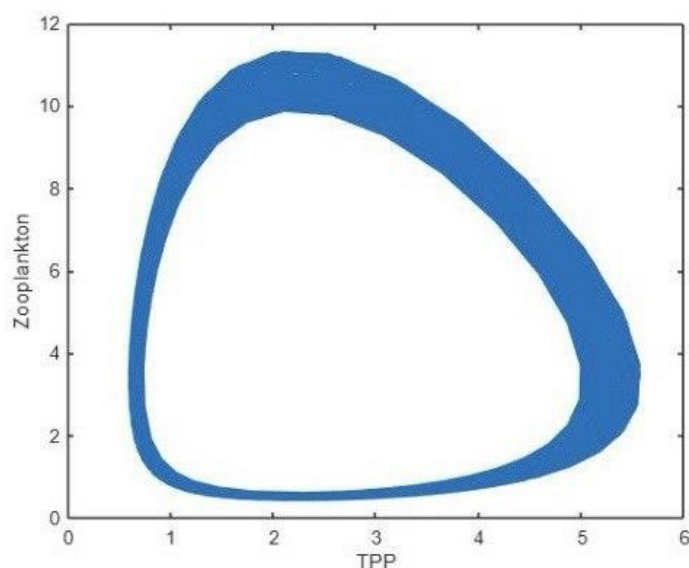


Fig. 10. Equilibrium between the TPP and zooplankton population loses its stability for $\beta = 0.7$

From numerical simulations, Figure 11 illustrates the simulation results of system for $\beta = 0.9$. The system is unstable as in the Figure 11. The TPP are almost completely protected, so zooplankton cannot regulate TPP. Therefore, the system illustrates that HAB phenomena will occur faster, thus making the system unstable. Figure 12 depicts the equilibrium between TPP and zooplankton. This shows that the prey-predator interaction exists between TPP and zooplankton population. The model shows large oscillation that indicate strong instability. This scenario represents a full of outbreak of HAB where ecological control fails due to uncontrolled TPP growth.

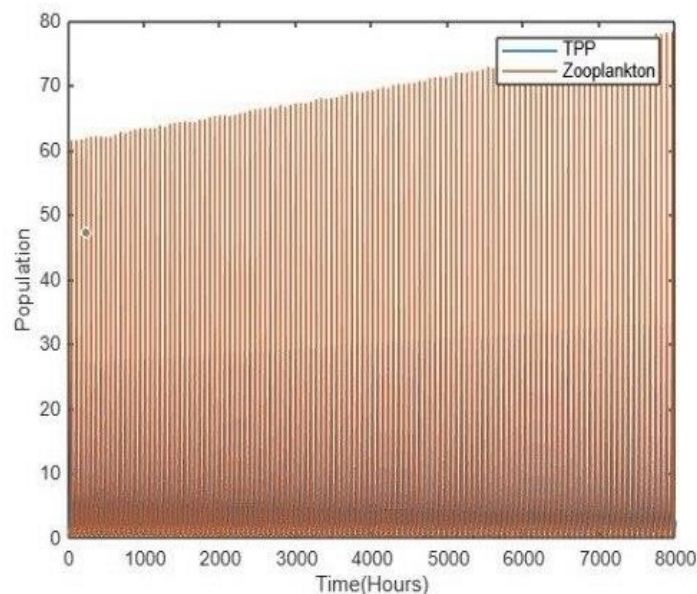


Fig. 11. Simulations results of system (1) with $\beta = 0.9$

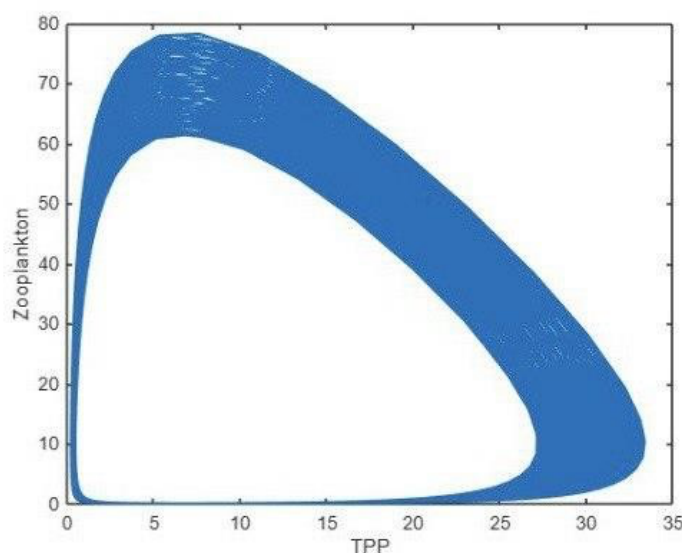


Fig. 12. Equilibrium between the TPP and zooplankton population loses its stability for $\beta = 0.9$

4. Conclusion

This study presented a mathematical model that describes the process of HAB occurrence. This study proposed a prey-predator model to investigate the harmful algal bloom (HABs) through the interaction between toxic-phytoplankton (TPP) and zooplankton populations. The model is derived based on prey-predator model where toxic-phytoplankton acts as prey and zooplankton as predators. Numerical simulations supported the analytical results that to observe system behaviours under different parameter conditions, particularly varying levels of prey refuge. However, the system transitions from stable to unstable as the rate of refuge prey increases. The result was observed when parameter of refuge prey is low, the system is stable and allows for balanced prey-predator interaction. As the value of the parameter of refuge prey increase, the capacity of zooplankton to control primary production TPP decreases, resulting in instability within the system. There was

observed that high refuge rates contributed to unstable dynamic and higher risks of harmful algal blooms.

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