Semarak Engineering Journal 8, Issue 1 (2025) 13-22

Semarak Engineering Journal

Journal homepage:
https://semarakilmu.my/index.php/sej/index
ISSN: 3036-0145

Coupled Numerical Approach for Combined Mode of Heat Transfer

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ARTICLE INFO

Article history:
Received 28 November 2024
Received in revised form 3 January 2025
Accepted 10 February 2025
Available online 20 March 2025

Keywords:
Conduction; Radiation; LBM; CVFEM; Participating medium; heat transfer; RTE

ABSTRACT

In this paper, we propose a two-dimensional transient coupled nonlinear conduction radiative heat transfer problem in an optically emitting, absorbing and scattering medium is proposed in this paper. The hybrid mathematical proposed model is based on the lattice Boltzmann method (LBM) in combination with the Control Volume Finite Element Method (CVFEM). On a coupled LBM-CVFEM need, the lattice Boltzmann method was used to solve the energy equation. The Radiative Transfer Equation (RTE) is coupled with the nonlinear heat conduction equation where the CVFEM has been adopted as the numerical technique for the radiative information required as a source in the energy equation. To study the compatibility and suitability of the LBM for the solution of the energy equation and the CVFEM for the radiative information, results were analyzed for the effects of various influencing parameters. Comparison with results of other methods validates the new model.

1. Introduction

Transient radiative conductive heat transfer is emerging as a major development in different energy engineering applications such as heat pipe, optical textile fiber processing, solar systems, thermo-physical properties measurement, multi layered insulations, glass fabrication, heat transfer through the semi-transparent, porous materials, industrial furnaces, fibrous insulation [1-4].

The constitutive medium in the majority of these energy engineering systems actively participates in the radiative transfer due to the absorption, emission and scattering of radiation where simulating the coupled radiative and conductive phenomena of heat transfer in the participating medium is a crucial task.

The D2Q9 lattice Boltzmann method (LBM) has evolved as an alternative numerical approach for the solution of a large class of problems [1-6,12-15] in the last decades. This numerical method has achieved excellent success in different engineering areas such as multiphase flow and complex fluid phenomena [5]. So, the lattice Boltzmann method which has the potential to become a versatile CFD

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https://doi.org/10.37934/sej.8.1.1322b

platform that is superior to the existing continuum based CFD methods is used for computing the diffusive part of radiative-diffusive heat transfer.

The objective of the present work is to establish the compatibility and the performance of the LBM for the solution of the energy equation and the CVFEM for the determination of radiative information. For radiative information, the CVFEM has been demonstrated to be successful in the solution of multidimensional enclosures, as well as for axisymmetric and non-axisymmetric radiative problems, and also for the solution of combined mode heat transfer in participating media [8]. CVFEM has the potential to become a versatile and very promising approach for the solution of radiative transfer problems in structured or unstructured meshes in multidimensional complex geometries, and this is in the presence of an isotropic or any anisotropic medium [9,10]. So, the proposed coupled numerical method will be a consistent numerical tool in order to achieve excellent success in different multi-mode engineering areas such as multiphase flow and complex fluid phenomena. To that end, a benchmark problem dealing with transient conduction radiation heat transfer in a 2-D rectangular enclosure is considered. Results of the LBM-CVFEM and the literature's results are compared against each other and a good concordance is highlighted. Besides, the effects of the scattering albedo parameter are studied.

2. Formulation

In the absence of convection and heat generation, the governing equation of a nonlinear coupled transient conduction and radiation problem can be written in the following form

$$\rho c_p \frac{\partial T}{\partial t} = k\nabla^2 T - \nabla.\vec{q}_R \quad (1)$$

Where ρ is the density, c_p is the specific heat and k is the thermal conductivity.

$\vec{q}_R$ represents the radiative heat flux which is given by

$$\vec{q}_R = \int_{4\pi} I\vec{\Omega}\, d\Omega \quad (2)$$

Where I is the radiative intensity which can be obtained by solving the Radiative Transfer Equations (RTE). For the RTE an absorbing, emitting and scattering grey medium can be written as

$$\vec{\nabla}.(I(s,\vec{\Omega})\vec{\Omega}) = -(k_a + k_d)I(s,\vec{\Omega}) + k_a I_b(s) + \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\vec{\Omega'})\Phi(\vec{\Omega'} \to \vec{\Omega})d\Omega' \quad (3)$$

where $I(s,\vec{\Omega})$ is the radiative intensity, which is a function of position s and direction $\vec{\Omega}$; k_a and k_d are absorption and scattering coefficients, respectively; $I_b(s)$ is the blackbody radiative intensity at the temperature of the medium; and $\Phi(\vec{\Omega'} - \vec{\Omega})$ is the scattering phase function from the incoming $\vec{\Omega'}$ direction to the outgoing direction $\vec{\Omega}$. The term on the left-hand side represents the gradient of the intensity in the direction $\vec{\Omega}$. The three terms on the right-hand side represent the changes in intensity due to absorption and out-scattering, emission, and in-scattering, respectively. The radiative boundary condition for Eq. (3), when the wall bounding the physical domain is assumed grey and emits and reflects diffusely, can be expressed as

$$I_w(\vec{\Omega}) = \frac{\varepsilon_w \sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi} \int_{\vec{\Omega'}.\vec{n}_w<0} I_w(\vec{\Omega'}) \left| \vec{\Omega'}.\vec{n}_w \right| d\Omega' \quad \text{if} \quad \vec{\Omega'}.\vec{n}_w > 0 \tag{4}$$

$\vec{n}_w$ is the unit normal vector on the wall and ε_w represents the wall emissivity.

2.1 The CVFEM for Radiative Information

The CVFEM is used to discretize the RTE. In the CVFEM, the spatial and angular domains are divided into a finite number of control volumes and control solid angles.

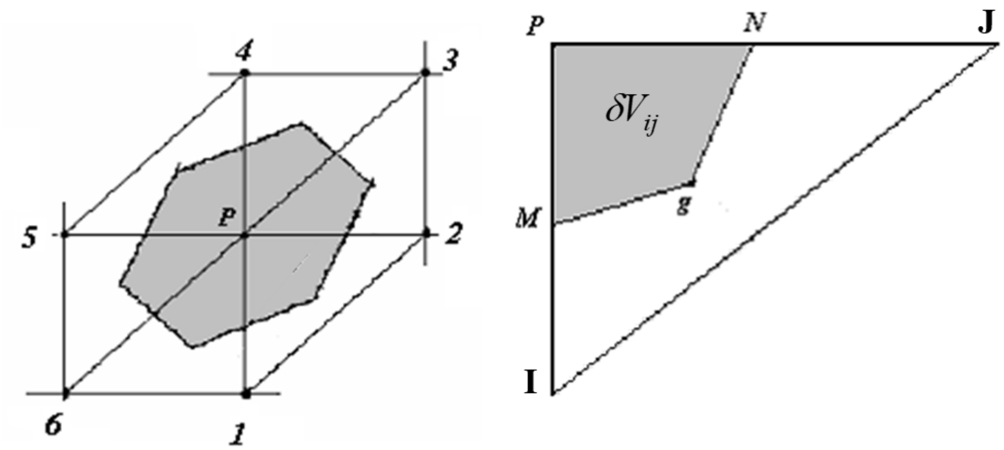
2.1.1 Angular discretization

The direction of propagation $\vec{\Omega}$ is defined by the couple (θ_p, φ) where θ_p and φ are, respectively, the polar and azimuthal angles and it is expressed as

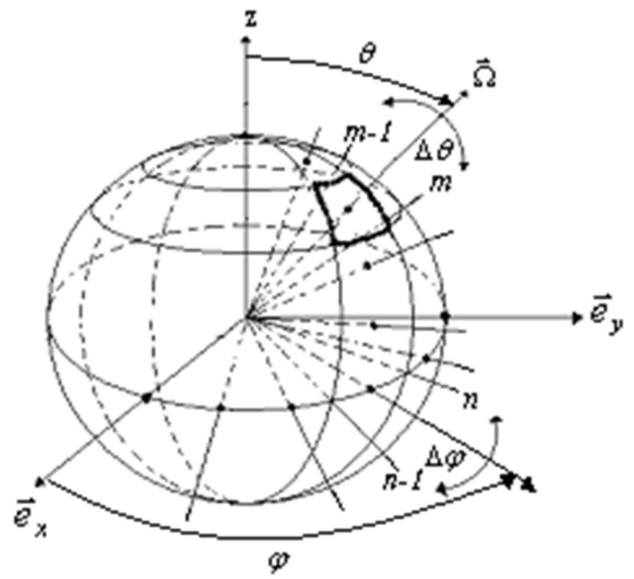
$$\vec{\Omega} = \sin\theta_p \cos\varphi \vec{e_x} + \sin\theta_p \sin\varphi \vec{e_y} + \cos\theta_p \vec{e_z} \tag{5}$$

The total solid angle is subdivided into $N_{\theta_p} \times N_\varphi$ control solid angles as depicted in Figure 1, where $\Delta\varphi = (\varphi^+ - \varphi^-) = 2\pi / N_\varphi$ and $\Delta\theta_p = (\theta_p{}^+ - \theta_p{}^-) = \pi / N_{\theta_p}$. The N_φ and $N_{\theta p}$ represent numbers of control angles in the polar and azimuthal directions, respectively. These $N_\varphi N_{\theta p}$ control solid angles are non-overlapping, and their sum is 4π. The control solid angle $\Delta\Omega^{mn}$ is given by (Figure 1)

$$\Delta\Omega^{mn} = \int_{\Delta\theta_p} \int_{\Delta\varphi} \sin\theta_p \, d\theta_p \, d\varphi \tag{6}$$



(a) (b)



(c)

Fig. 1. (a) Spatial discretization in ($\vec{e_x},\vec{e_y}$) plan, (b) Control volume ΔV_{ij}, (c) Subvolume cross section in ($\vec{e_x},\vec{e_y}$) plan

2.1.2. Spatial discretization

The spatial domain is subdivided into three-node triangular elements. As shown in Figure 1, a control volume ΔV_{ij} is created around each node N by enjoining the centroids G_l of the elements to midpoints M_l and M_{l+1} of the corresponding sides. Each element has two faces, M_lG_l and G_lM_{l+1}; bounding the sub-control volume around N; and each control volume is constructed by adding all sub volumes $NM_lG_lM_{l+1}N$ (Figure 1). The obtained mesh is composed of N_xN_y control volumes ΔV_{ij}. The N_x and N_y represent numbers of nodes in x and y direction, respectively. Δx and Δy represent the regular steps in x and y direction, respectively, and they are given by: $\Delta x = L_x/(N_x-1), \Delta y = L_y/(N_y-1)$ with L_x and L_y are, respectively, the x and y dimensions of the calculation domain (Figure 1).

2.1.3. Discretized RTE

Integrating Eq. (3) over the control volume ΔV_{ij} (Figure 1) and the control solid angle $\Delta\Omega^{mn}$ (Figure 1), we obtain

$$\begin{aligned}\int_{A^N}\int_{\Delta\Omega^{mn}} I(s,\vec{\Omega}).\vec{\Omega}.\vec{n_N}\,d\Omega dA = -\int_{\Delta V_{ij}}\int_{\Delta\Omega^{mn}}(k_a+k_d)I(s,\vec{\Omega})d\Omega dV \\ +\int_{\Delta V_{ij}}\int_{\Delta\Omega^{mn}} k_a I_b(s)d\Omega dV + \int_{\Delta V_{ij}}\int_{\Delta\Omega^{mn}}\frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\vec{\Omega'})\Phi(\vec{\Omega'}\to\vec{\Omega})d\Omega' d\Omega dV\end{aligned} \quad (7)$$

where A^N represents the surface of the control volume ΔV_{ij}.

In order to approximate the integrals that represents the extinction; emission and in-scattering contributions, the radiation intensity is considered constant within ΔV_{ij} and $\Delta\Omega^{mn}$ an is evaluated at the centroid of the control volume and at the centre direction of the control solid angle. For the term on the left-hand side in Eq. (7), the divergence theorem, the Skew Positive Coefficient Upwind (SPCU) interpolation scheme [8], and step schemes are used to calculate the corresponding expression. The final algebraic equation of the RTE is given by the following formulation

$$\gamma_{1ij}^{mn} I_{ij-1}^{mn} + \gamma_{2ij}^{mn} I_{i+1j}^{mn} + \gamma_{3ij}^{mn} I_{i+1j+1}^{mn} + \sum_{(m',n')=(1,1)}^{(N_\theta,N_\varphi)} \alpha_{ij}^{mnm'n'} I_{ij}^{m'n'} + \gamma_{4ij}^{mn} I_{ij+1}^{mn} + \gamma_{5ij}^{mn} I_{i-1j}^{mn} + \gamma_{6ij}^{mn} I_{i-1j-1}^{mn} = \beta_{ij}^{mn} \quad (8)$$

Where I, are the radiative intensities.

Then the algebraic Eq. (8) can be written in the following matrix form

$$AI = b \quad (9)$$

The obtained matrix system can be by the use of many iterative methods employed in CFD such as Conjugate Gradient methods, Lanczos method, and Jaccobi method. In the present work, the obtained matrix system is solved using the conditioned conjugate gradient squared method (CCGS). Once the intensity distributions are known, radiative information $\nabla.\vec{q}_R$ required for the energy equation is computed from

$$\nabla.\vec{q}_R = k_a(4\sigma T^4 - \int_{4\pi} I(\Omega)d\Omega) \quad (10)$$

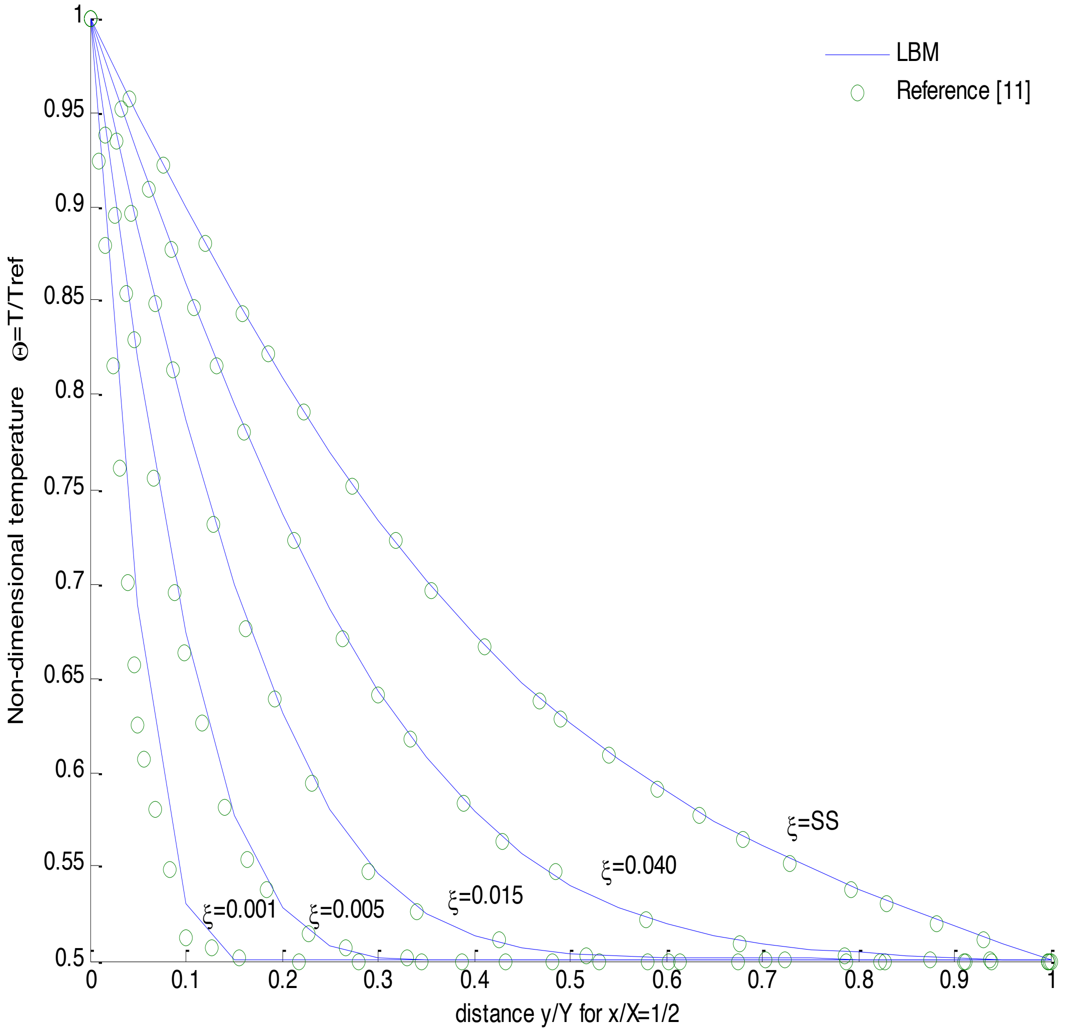
2.2 LBM for Energy Equation

The starting point of the LBM is the kinetic equation which for a 2-D geometry is given by [1-7]

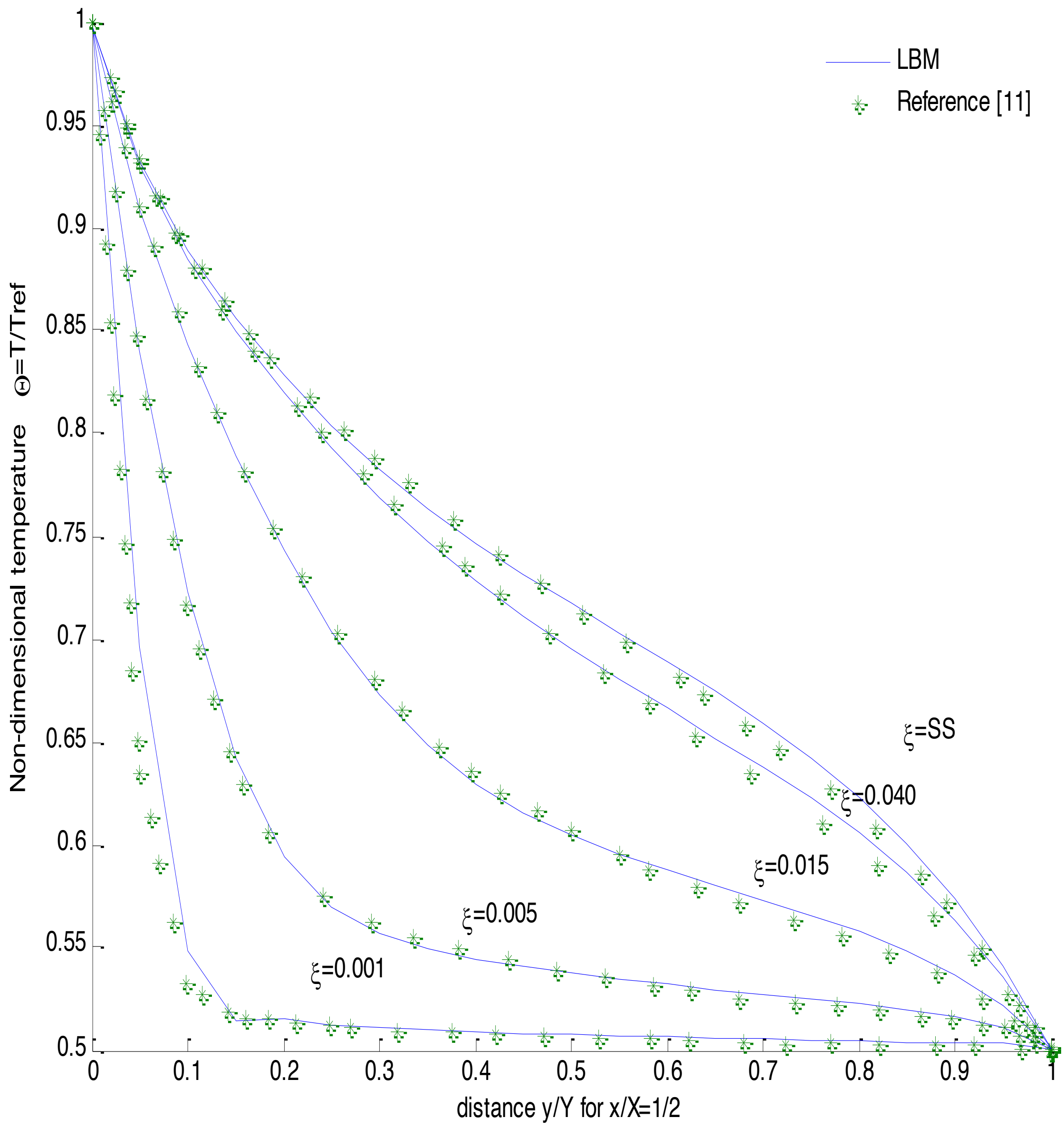
$$\frac{\partial f_i(\vec{r},t)}{\partial t} + \vec{c}_i.\nabla f_i(\vec{r},t) = \Omega_i, \qquad i = 1,2,3,\ldots,8 \quad (11)$$

where f_i is the particle distribution function denoting the number of particles at the lattice node $\vec{r} = (\vec{r}(x,y))$ and time t moving in direction i with velocity $\vec{c_i}$ along the lattice link $\Delta\vec{r} = \vec{c_i}\Delta t$ connecting the nearest neighbours and b is the number of directions in a lattice through which the information propagates. The term Ω_i represents the local change in f_i due to particle collisions. Using the single time relaxation model of the Bhatanagar–Gross–Krook (BGK) approximation, the discrete Boltzmann equation in the presence of volumetric radiation is given by [1-7].

$$f_i(\vec{r} + \vec{c_i}\Delta t, t + \Delta t) = f_i(\vec{r},t) - \frac{\Delta t}{\tau}[f_i(\vec{r},t) - f_i^{(0)}(\vec{r},t)] - (\frac{\Delta t}{\rho c_p}) w_i \nabla.\overrightarrow{q_R}(\vec{r},t) \quad (12)$$


LBM
Reference [11]
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
Non-dimensional temperature Θ=T/Tref
ξ=SS
ξ=0.040
ξ=0.015
ξ=0.005
ξ=0.001
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
distance y/Y for x/X=1/2

(a)

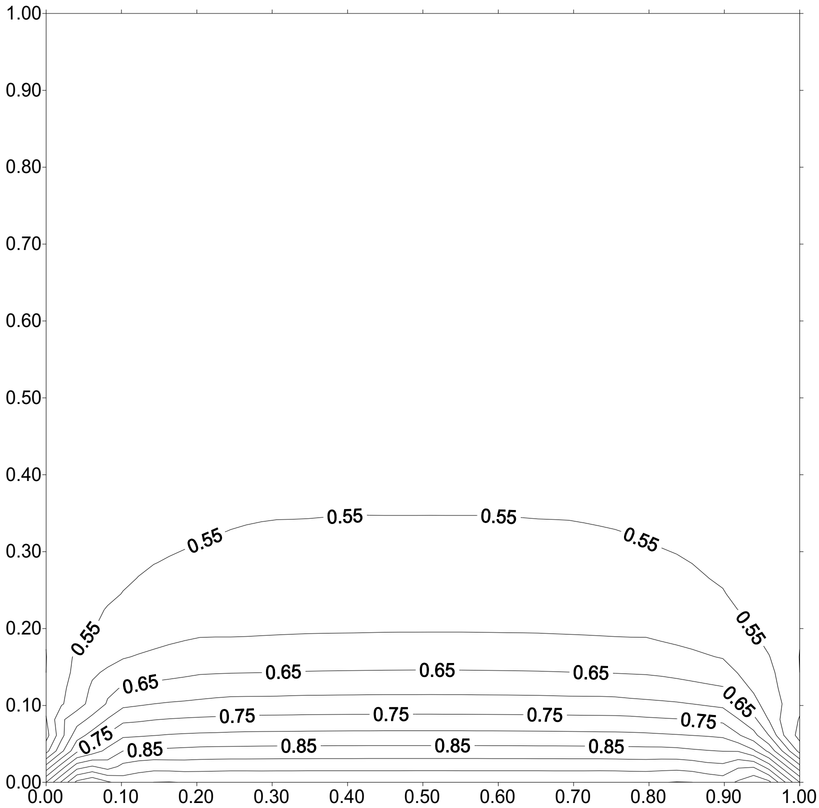


(b)

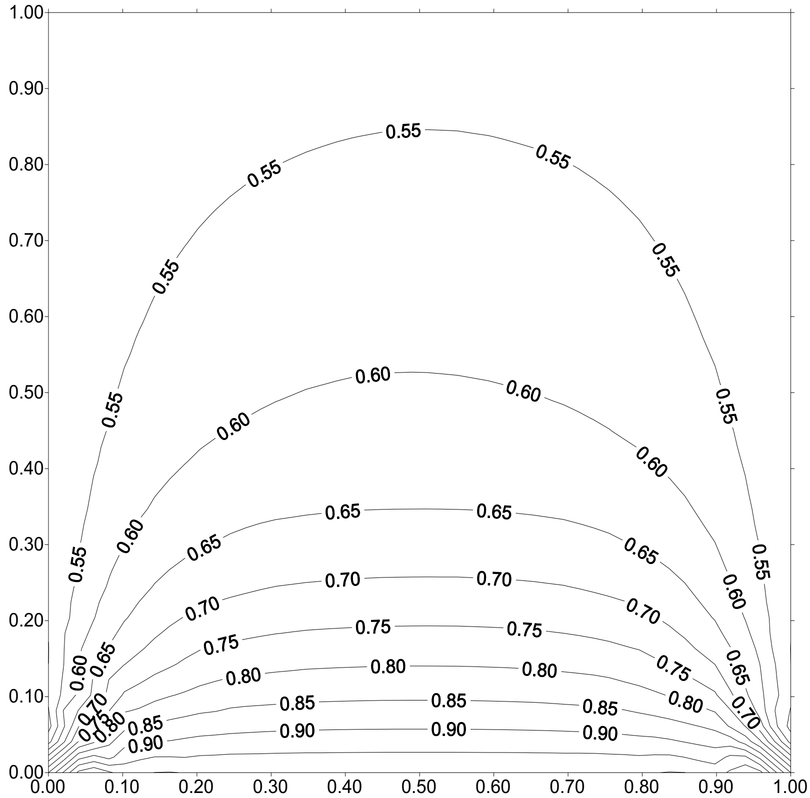
Fig. 2. Comparison of non-dimensional centerline temperature T/T_S in a 2-D square enclosure at different instants ξ, (a) $\omega = 0.0$ $\beta = 1.0$ $N = 0.01$,(b) $\omega = 0.5$ $\beta = 1.0$ $N = 0.01$

3. Results and Discussion

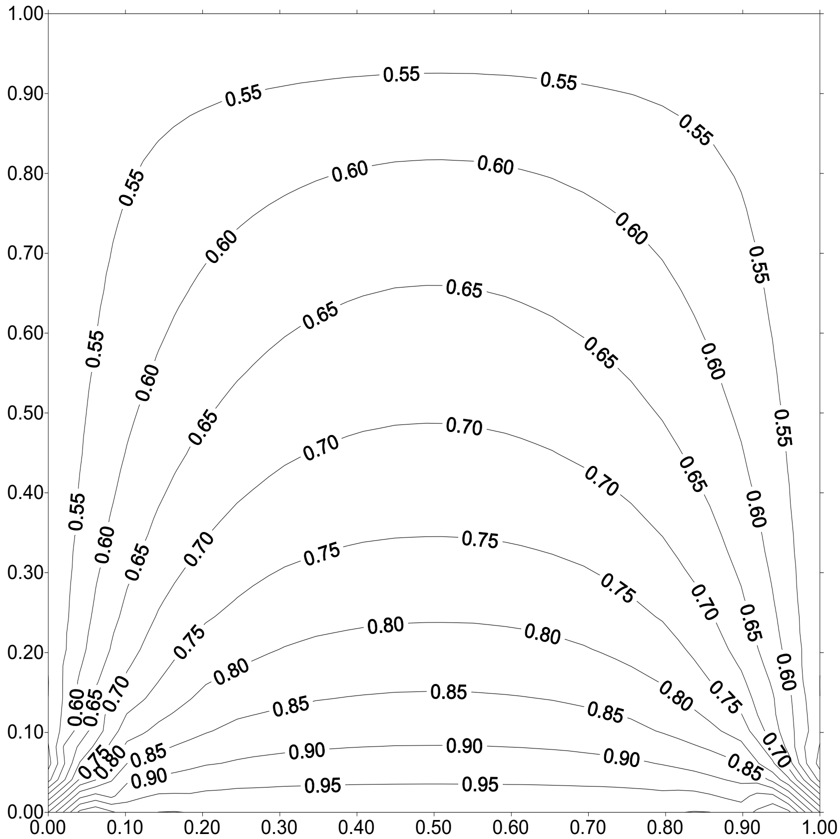
Transient conduction and radiation heat transfer in a 2-D square enclosure is considered. In this, initially, the entire system is at temperature $T_i = T_N = T_W = T_E$. For $t > 0$ the south boundary temperature is raised to $T_S = 2T_i$. The enclosed grey-homogeneous medium is absorbing, emitting and isotropically scattering. We investigate first the effect of the scattering albedo conduction radiation parameter. Comparison of the LBM–CVFEM and the LBM–CDM [11] temperature results for the conduction–radiation parameter values $N = 0.01$ are shown in Figure 2(a)-(b). Results are obtained for an extinction coefficient $\beta = 1.0$, a scattering albedo $\omega = 0.0$ (weak scattering Fig. 2-a) and $\omega = 0.5$ (a scattering comparable to absorption Figure 2(b) with black boundaries. It is seen from these figures that at all instants ξ, LBM–CVFEM and reference's results are in good agreement. In Figure 3 for an extinction coefficient $\beta = 1.0$, a scattering albedo $\omega = 0.5$ and conduction–radiation parameter $N = 0.01$, the spatial evolution of the isotherms distribution is plotted at different instants ξ.



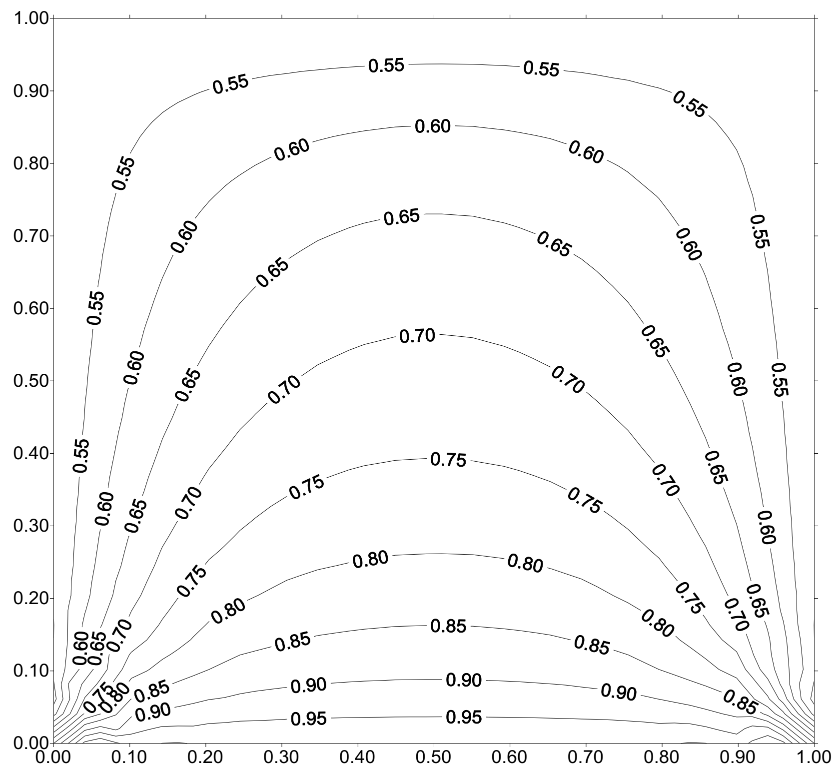
(a)



(b)



(c)



(d)

Fig. 3. Evolution of isotherms for (a) $\xi = 0.005$, (b) $\xi = 0.015$, (c) $\xi = 0.040$ and (d) $\xi = \infty$

4. Conclusions

A two-dimensional nonlinear coupled transient conduction and radiation heat transfer problem is solved via a new hybrid algorithm. The lattice Boltzmann method (LBM) was used to solve the energy equation of transient conduction–radiation heat transfer problem in a 2-D square geometry containing an absorbing, emitting and scattering medium. The radiative source term in the energy equation was computed using the control volume finite element method (CVFEM). The results highlight the robustness of LBM coupled to the CVFEM as an efficient energy equation solver, especially for problems with complex multidimensional with grey or non-grey energy engineering applications.

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