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Magnetohydrodynamic of Williamson Hybrid Nanofluids Flow over a Non-Linear Shrinking Sheet with Viscous Dissipation and Joule Heating

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ABSTRACT

Heat transfer plays a crucial role in various industrial applications. Thus, this study investigates the heat transfer characteristics of a non-Newtonian Williamson hybrid nanofluids flowing over a non-linear shrinking sheet, incorporating MHD effects and viscous dissipation. Alumina and Copper nanoparticles are dispersed in a CMC-water base fluid, representing a non-Newtonian hybrid nanofluid with shear thinning behaviour. The complex mathematical model is transformed into similarity equations using appropriate transformations, and the MATLAB function bvp4c is employed to solve these equations numerically. The model's accuracy is validated by comparison with an established model, demonstrating reasonable agreement. The study analyses the impact of various fluid parameters, including magnetic, Eckert number, Williamson, suction, and nanoparticle volume fraction, on fluid flow behaviour. Results show that increased suction enhances both the skin friction coefficient and heat transfer rate, while a higher Williamson parameter reduces both. The heat transfer rate decreases with an increase in the Eckert number. Additionally, an increase in the magnetic parameter and nanoparticle volume fraction leads to higher skin friction but a lower heat transfer rate.

1. Introduction

In the industrial process and manufacturing, the use of an effective working fluid is essential for achieving optimal production efficiency. In 1995, a breakthrough occurred with the introduction of Nanofluids (NFs) which consist of nanoparticles dispersed in a base fluid, as developed by Choi and Eastman [1]. These NFs have demonstrated superior performance compared to traditional fluids in

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various heat transfer applications, particularly in energy-intensive systems. Common base fluids include water and oil, while frequently used nanoparticles, such as Alumina (Al_2O_3) and Copper (Cu), are valued for their stability and high thermal conductivity, respectively. Numerous studies have been conducted to investigate the behavior of NFs [2-7], with recent research highlighting that factors such as nanoparticle type, concentration, and base fluid selection significantly enhance heat transfer rates [8].

The development of hybrid nanofluids (HNFs), achieved by adding a second type of nanoparticle to conventional NFs, has further improved their thermal properties. Pioneering experimental studies in this field include those by Turcu *et al.,* [9], Jana *et al.,* [10], and Suresh *et al.,* [11]. Since experimental work involves material preparation, which needs high financial support and even involves certain procedures which may harm humans and the environment, several researchers look for numerical approaches to overcome these challenges. Notable contributions in this area include those by Devi and Devi [12], Takabi and Salehi [13], and Xue *et al.,* [14] who developed different thermophysical correlations by implementing the Tiwari Das model of NFs to explore the heat transfer characteristics of HNFs numerically. These correlations have since been widely used to study HNFs' flow behavior over various geometries and under different conditions [15-27].

Depending on nanoparticle volume fractions and the type of base fluid, HNFs may exhibit non-Newtonian behavior characterized by complex rheological properties. Traditional Navier-Stokes equations are often inadequate for modeling such fluids, which can exhibit shear-thinning or shear-thickening behavior. In this respect, various non-Newtonian models have been employed in HNFs studies, including Casson, Maxwell, Viscoelastic, Reiner-Philippoff, and Williamson models [28-32].

Among these, the Williamson model is particularly noteworthy as it effectively represents pseudoplastic fluids, which are the most encountered non-Newtonian fluid [33]. Pseudoplastic fluids, such as polymer solutions, paint, blood, and plasma, are shear-thinning whose viscosity decreases with increasing shear stress. The Williamson model accounts for both the minimum and maximum viscosities of pseudoplastic fluids, providing more accurate results and fitting experimental data effectively [34]. Consequently, many recent studies have incorporated the mathematical formulation of HNFs with the Williamson fluid model, resulting in what is known as the Williamson hybrid nanofluids (WHNFs) model, particularly for investigating fluids with shear-thinning characteristics [32,35-38]. Most previous WHNFs studies focused on fluid flow induced by a linear velocity. However, as emphasized by Timol [39], for non-Newtonian fluids, a non-linear velocity in the form of a power law ($x^{1/3}$) should be adopted. This power law form of velocity has been explored in several studies [24, 40-43].

In the field of fluid mechanics, Crane [44] and Wang [45] were pioneers in describing the flow behavior over stretching and shrinking surfaces, respectively. These surfaces have drawn significant attention from scholars due to their diverse and important applications in technology and industry, such as wire drawing, aerodynamic extrusion of plastic sheets, hot rolling, and metal spinning. Miklavčič and Wang [46] later revealed that fluid flow solutions over a shrinking sheet are not unique. Multiple solutions have also been identified in various types of fluid flows over shrinking surfaces, including HNFs [47-49], Williamson fluid [50] and Williamson fluid with nanoparticles [51,52].

The application of magnetohydrodynamics (MHD) to fluid flow has gained importance in various fields, including nuclear reactors, plasma confinement, and metallurgical processes. MHD governs the behaviour of electrically conducting fluids in the presence of a magnetic field, where Lorentz forces influence velocity profiles and heat transfer. Considering its wide-ranging applications, numerous researchers have studied MHD HNFs [16,18,40,53-60] and MHD Williamson fluids [61,62] for various geometries and flow conditions. Kavya *et al.,* [37], Almaneea [63], Yahya *et al.,* [64], and Alkasasbeh *et al.,* [65], investigated WHNFs involving MHD effects but with different geometries, nanoparticles and base fluids. Kavya *et al.,* [37] studied magnetic hybrid nanoparticles in a water suspension of MoS_4 and Cu nanoparticles for flow over stretching/shrinking cylinder. Almaneea [63], explored MHD HNFs composed of Al_2O_3 and Cu nanoparticles with glycerine as the base fluid for flow over a heated pipe. Kavya *et al.,* [37], and Almaneea [63] reported that as the magnetic field strength increased, the velocity profile decreased while the temperature increased. This finding consistent with Shateyi *et al.,* [61], and Hussain *et al.,* [62]. In contrast, Khashi'ie *et al.,* [40] observed the opposite trend, where the velocity increased, and the temperature decreased with an increasing magnetic parameter. Yahya *et al.,* [64] analysed the thermal performance of an engine oil-based HNF consists of Go and AA7072 nanoparticles, across a Riga wedge, while Alkasasbeh *et al.,* [65] investigated MHD WHNFs composed of SWCNTS and MWCNTS with water as the base fluid over an exponentially shrinking sheet. Recently, Ali *et al.,* [66] studied an MHD Cross ternary HNF containing MoS_2, TiO_2 and Ag with CMC-water-base fluid over a stretching cylinder. CMC-water has emerged as a popular base fluid for stabilizing HNFs [67]. Experimental results show that CMC-water exhibits shear thinning behavior, and the outcomes align well with the power-law model for non-Newtonian fluids. Recent research on MHD WHNFs flow can be found in the work of Jain *et al.,* [68], and Aselebe *et al.,* [69] focusing on viscous dissipation and Joule heating effects, respectively. These effects are particularly intriguing due to their significant impact on MHD fluid flow. Jain *et al.,* [68] compared nanofluids such as Cuo-water, SWCNT-water, and MWCNT-water and found that hybrid carbon nanotubes demonstrated superior performance in terms of skin friction and local Nusselt number compared to SWCNT-water and MWCNT-water. Meanwhile, Aselebe *et al.,* [69] reported an increase in fluid temperature due to viscous dissipation.

From the literature, there is limited research on MHD WHNFs under the combined influences of viscous dissipation and Joule heating over a shrinking sheet induced by a non-linear velocity. This study, therefore, investigates the behaviour of MHD WHNFs flow over a non-linear shrinking sheet, incorporating the effects of viscous dissipation and Joule heating. Alumina and Copper nanoparticles are suspended in a CMC-water base fluid to represent a non-Newtonian hybrid nanofluid. The existing formulation of HNFs is integrated with the Williamson fluid model. Due to the complexity of the governing equations describing fluid flow and heat transfer, the mathematical model is reduced to a simplified set of ordinary differential equations (ODEs) using a similarity transformation. These equations are then solved numerically using the bvp4c function in MATLAB. The results are presented graphically, and the effects of various fluid parameters, including the magnetic parameter, Eckert number, Williamson parameter, Prandtl number, and suction parameter, on the velocity and temperature profiles, as well as physical quantities like skin friction coefficient and Nusselt number, are analysed to elucidate fluid flow behaviour. Additionally, the simultaneous impact of the Williamson parameter and the volume fraction of nanoparticles on heat transfer enhancement is also investigated.

2. Methodology

The physical flow model of WHNFs over a shrinking sheet is illustrated in Figure 1. The surface velocity is described by $u_w(x) = ax^{1/3}$ where a is a constant and $a > 0$. The parameter $v_w(x)$ represents the mass flux velocity, while $T_w(x) = T_\infty + T_0 x^{2/3}$ is treated as a variable surface temperature. Here, T_∞ and T_0 denote the ambient and constant temperatures, respectively. A magnetic field, $B(x)$, is applied transversely along the y-axis, where $B(x) = B_0 x^{-1/3}$ and B_0 represents the constant magnetic strength [40]. Additionally, the effects of viscous dissipation and Joule heating are considered in the analysis.

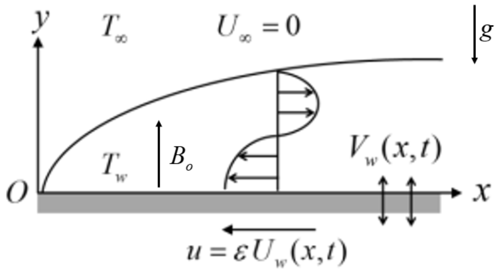

Fig. 1. Geometry of the physical problem

The governing equations for WHNFs can be derived using boundary layer approximations to the continuity, momentum, and energy equations. Consequently, the steady two-dimensional boundary layer equations for this fluid model are expressed as follows [15,24,38,70]:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\Gamma\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}\right) - \frac{\sigma_{hnf}}{\rho_{hnf}}B_0^2 u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}}\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{hnf}}{(\rho c_p)_{hnf}}\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^3\right\} + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}}B_0^2 u^2 \tag{3}$$

In this context, (x, y) represents the Cartesian coordinates, while (u, v) denote the velocity components in the x- and y-directions, respectively. Additionally, k_{hnf}, ρ_{hnf}, μ_{hnf}, $(c_p)_{hnf}$, $(\sigma)_{hnf}$, Γ, and g correspond to the thermal conductivity, density, dynamic viscosity, heat capacitance, electrical conductivity, time constant, and gravitational acceleration, respectively. The problem is governed by the following boundary conditions:

$$\begin{aligned} &u = \varepsilon u_w(x), \quad v = v_w(x), \quad T = T_w(x) \quad \text{at } y = 0, \\ &u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

where ε represents the deformable sheet, where $\varepsilon > 0$ indicates a stretching sheet, $\varepsilon < 0$ denotes a shrinking sheet and $\varepsilon = 0$ corresponds to a static sheet. Additionally, throughout this section, the subscripts *hnf* and *f* refer to hybrid nanofluid and regular fluid respectively.

The governing Eq. (1)-(3) are expressed as nonlinear partial differential equations (PDEs). Due to their complexity, the similarity transformation method is applied to simplify them into nonlinear ordinary differential equations (ODEs). According to references [59,68], the appropriate similarity variables are introduced in Eq. (5) as follows:

$$\eta = \frac{y}{x^{1/3}}\left(\frac{a}{\nu_f}\right)^{1/2}, \quad \psi = \left(a\nu_f\right)^{1/2} x^{2/3} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where η and θ are dimensionless similarity variables related to the stream function ψ. Utilizing these variables, and considering that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, the velocity components are transformed into:

$$u = a x^{1/3} f'(\eta),\ v = -\left(a\nu_f\right)^{1/2} x^{-1/3}\left(\frac{2}{3} f(\eta) - \frac{1}{3}\eta f'(\eta)\right) \tag{6}$$

which satisfied the continuity Eq. (1). Additionally, $v_w = -(2/3)\left(a\nu_f\right)^{1/2} x^{-1/3} S$ represents the mass velocity at the surface, where ν_f represents the fluid's kinematic viscosity and S is the suction/injection parameter. A positive S corresponds to suction, while a negative S indicates injection.

Subsequently, Eq. (5) and (6) are substituted into the governing Eq. (2) and (3) to derive the transformed ODEs, which are presented as Eq. (7) and (8):

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left(1 + \gamma f''\right) f''' + \frac{2}{3} f f'' - \frac{1}{3} f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M f' = 0 \tag{7}$$

$$\frac{1}{\Pr}\left(\frac{k_{hnf}}{k_f}\right)\theta'' + \left(\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}\right)\left(\frac{2}{3} f'\theta + \frac{2}{3} f\theta'\right) + \left(\frac{\mu_{hnf}}{\mu_f}\right) Ec f''^2\left[1 + \frac{\gamma}{2} f''\right] + \left(\frac{\sigma_{hnf}}{\sigma_f}\right) Ec M f'^2 = 0 \tag{8}$$

These equations are subject to the corresponding boundary conditions given by:

$$\begin{gathered} f(0) = S,\ f'(0) = \varepsilon,\ \theta(0) = 1 \\ f'(\eta) \to 0,\ \theta(\eta) \to 0 \ \text{ as } \eta \to \infty \end{gathered} \tag{9}$$

In this context, the prime notation $(\ )'$ denotes differentiation with respect to η. The dimensionless parameters associated with the Williamson fluid (γ), magnetic field (M), Eckert number (Ec), and Prandtl number $(\Pr)$ are defined as follows:

$$\gamma = \Gamma a\left(\frac{2a}{\nu_f}\right)^{1/2},\ M = \frac{\sigma_f b_0^2}{\rho_f a},\ Ec = \frac{a^2}{T_o (c_p)_f},\ \Pr = \frac{(\rho c_p \nu)_f}{k_f}, \tag{10}$$

where $\mathrm{Re}_x = u_w(x) x/\nu_f$ represents the local Reynolds number.

Eq. (11) defines the physical quantities of interest, specifically the skin friction coefficient C_f and the local Nusselt number Nu_x:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left[\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0},\ Nu_x = -\frac{x k_{hnf}}{k_f\left(T_w - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

By substituting the variables from Eq. (5) into Eq. (11), it transforms into:

$$Re_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} \left(f''(0) + \frac{\gamma}{2} f''(0) \right), \quad Re_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{12}$$

In this study, the thermophysical correlations used to solve the HNFs flow problem are based on the model of Takabi and Salehi [13]. For clarity, the thermophysical correlations for both NFs and HNFs are detailed in Table 1. Throughout the analysis, a volume fraction of nanoparticles $(\phi_1 = \phi_2 = 0.01)$ is applied, resulting in Cu-Al_2O_3/CMC-water hybrid nanofluid. Note that, ϕ_1 and ϕ_2 represent the nanoparticles concentration for Al_2O_3 and Cu, respectively, and the summation of them represented by ϕ_{hnf}. Additionally, the thermophysical properties of the base fluid (CMC-water) and the nanoparticles (Cu, and Al_2O_3) are listed in Table 2 [71,72].

Table 1
Thermophysical properties for nanofluid and hybrid nanofluid

Element	Nanofluid	Hybrid nanofluid
Viscosity	$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$	$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1-\phi_{hnf})^{2.5}}$
Density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = (1-\phi_{hnf})\rho_f + \phi_1\rho_{s1} + \phi_2\rho_{s2}$
Heat capacity	$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$	$(\rho C_p)_{hnf} = (1-\phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_{s1} + \phi_2(\rho C_p)_{s2}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$	$\frac{k_{hnf}}{k_f} = \frac{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hnf}}\right) + 2k_{bf} + 2(\phi_1 k_{s1} + \phi_2 k_{s2}) - 2\phi_{hnf} k_{bf}}{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hnf}}\right) + 2k_{bf} - (\phi_1 k_{s1} + \phi_2 k_{s2}) + \phi_{hnf} k_{bf}}$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_s + 2\sigma_f - 2\phi(\sigma_f - \sigma_s)}{\sigma_s + 2\sigma_f + \phi(\sigma_f - \sigma_s)}$	$\frac{\sigma_{hnf}}{\sigma_f} = \frac{\left(\frac{\phi_1 \sigma_{s1} + \phi_2 \sigma_{s2}}{\phi_{hnf}}\right) + 2\sigma_{bf} + 2(\phi_1 \sigma_{s1} + \phi_2 \sigma_{s2}) - 2\phi_{hnf} \sigma_{bf}}{\left(\frac{\phi_1 \sigma_{s1} + \phi_2 \sigma_{s2}}{\phi_{hnf}}\right) + 2\sigma_{bf} - (\phi_1 \sigma_{s1} + \phi_2 \sigma_{s2}) + \phi_{hnf} \sigma_{bf}}$
		where $\phi_{hnf} = \phi_1 + \phi_2$

Table 2
Thermophysical properties for the base fluid and nanoparticles

Thermophysical properties	Base fluid	Nano particle	
	CMC-water	Al_2O_3	Cu
Density, $\rho\,(kg/m^3)$	997.1	3970	8933
Heat capacitance, $C_p (J/kgK)$	4179	765	385
Thermal conductivity, $k(W/mK)$	0.613	40	400
Electrical conductivity, $\sigma\,(S/m)$	0.05	0.85	1.67
Prandtl, Pr	6.2		

3. Results

Eq. (7) to (9) were solved using the numerical approach with MATLAB's bvp4c function. The effects of various physical parameters on the WHNFs flow behavior were analyzed by adjusting the control parameters accordingly. Prior to obtaining the solutions, a validation process was carried out to ensure the accuracy of the current model. Under specific limiting conditions, the momentum equation in this study was reduced to those found in previous works by Waini *et al.,* [24], Cortell [41], and Ferdows *et al.,* [42], as shown in Table 3. The accuracy of the current numerical method was confirmed when the values of $f''(0)$, as presented in Table 4, showed reasonable agreement with previous studies. These values were obtained under the condition of a stretching sheet $(\varepsilon = 1)$ for various values of S and certain limiting values.

Table 3
Comparative model in terms of momentum equations

Author	Model	Limiting cases
Current	$\frac{(\mu_{hnf}/\mu_f)}{(\rho_{hnf}/\rho_f)} f'''(1+\gamma f'') + \frac{2}{3} ff'' - \frac{1}{3} f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} Mf' = 0$	$\gamma = M = \phi_{hnf} = 0$
Waini *et al.,* [24]	$3\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + 2ff'' - f'^2 = 0$	
Cortell [41]	$3f''' + 2ff'' - (f')^2 = 0$	
Ferdows *et al.,* [42]	$f''' + \frac{2}{3} ff'' - \frac{1}{3}(f'^2 - Mf'^2) + Gr\theta + Gc\phi = 0$	$M = Gr = Gc = 0$

Table 4
Comparative values of $f''(0)$ for various values of S when $\mathbf{Pr} = 2$, $\gamma = M = \phi_{hnf} = 0$, and $\varepsilon = 1$

S	Cortel [41]	Ferdows *et al.,* [42]	Waini *et al.,* [24]	Current
0.75	-0.453521	-0.453523	-0.453523	-0.453526660
-0.5	-0.518869	-0.518869	-0.518869	-0.518871662
0	-0.677647	-0.677648	-0.677648	-0.677648605
0.5	-0.873627	-0.873643	-0.873643	-0.873642953
0.75	-0.984417	-0.984439	-0.984439	-0.984439416

The fluid flow of HNFs is characterized by 2% nanoparticle concentration $(\phi_1 = \phi_2 = 0.01)$ and is driven by a shrinking surface $(\varepsilon = -1)$. The solutions for the skin friction coefficient, $\mathrm{Re}_x^{1/2} C_f$ and the Nusselt number, $\mathrm{Re}_x^{-1/2} Nu_x$ for various parameter values are recorded in Table 5. An increase in the values of S enhances both $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$. In contrast, increasing the values of γ results in a decrease in these physical quantities. Additionally, the values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ increase and decrease, respectively, with the rise of M. Moreover, Ec and Pr have no effect on $\mathrm{Re}_x^{1/2} C_f$. However, these parameters influence the energy equation. As a result, the values of $\mathrm{Re}_x^{-1/2} Nu_x$ are affected, decreasing as Ec increase but increasing as Pr rise. Physically, the presence of suction and a higher Prandtl number tends to release energy to the flow, while the Williamson fluid parameter, magnetic parameter, and Eckert number act to hinder the flow's energy.

Table 5
Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for different physical parameters when $\varepsilon = -1, \phi_1 = \phi_2 = 0.01$.

S	γ	M	Ec	Pr	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$
2.25	0.01	0.01	0.1	6.2	1.110990647	7.822658227
2.24					1.092462312	7.776725260
2.22					1.050075988	7.683979925
2.25	0.02				1.106574516	7.821477794
	0.03				1.102116671	7.820313237
	0.04				1.097612236	7.819163763

0.01	0			1.080725411	7.830468736
	0.02			1.136413715	7.815525959
	0.03			1.158831679	7.808865051
	0.01	0		1.110990649	8.289691565
		0.2		1.110990647	7.355624890
		0.3		1.110990647	6.888591552
		0.1	7	1.110990649	8.958907313
			8	1.110990649	10.378906129
			10	1.110990667	13.217952059

The velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for the previously mentioned parameters are shown in Figures 2-9. The far-field boundary conditions were satisfied asymptotically. Figures 2 and 3 illustrate the effect of magnetic parameters M on the velocity $f'(\eta)$and temperature profile $\theta(\eta)$. It is observed that as this parameter increases, the velocity increases, whereas the temperature decreases along the shrinking surface. Theoretically, magnetic parameters may reduce the velocity due to the Lorentz force, but in the present study, the velocity increases. Figures 4 and 5 depict the influence of the Eckert number, Ec and the Prandtl number, $\Pr$ on the temperature profile $\theta(\eta)$, respectively. Both parameters lead to an increase in the temperature profile $\theta(\eta)$.

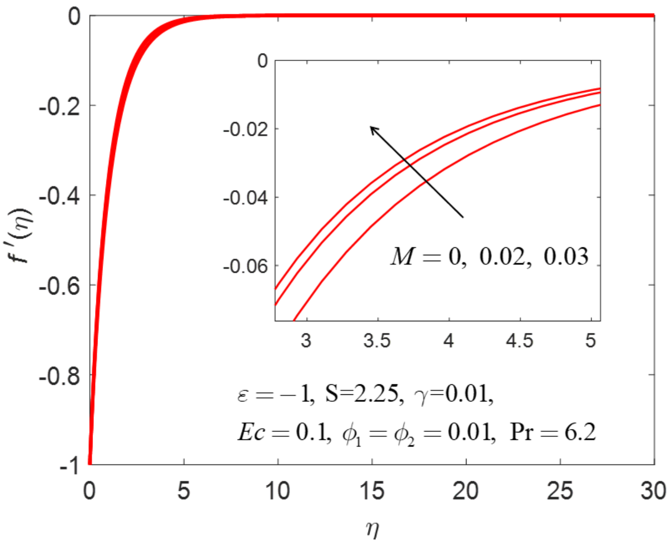

Fig. 2. $f'(\eta)$for different values of M

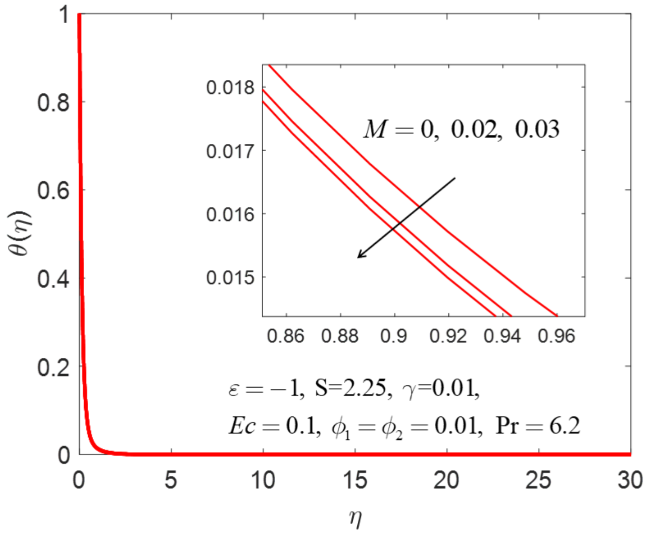

Fig. 3. $q(h)$for different values of M

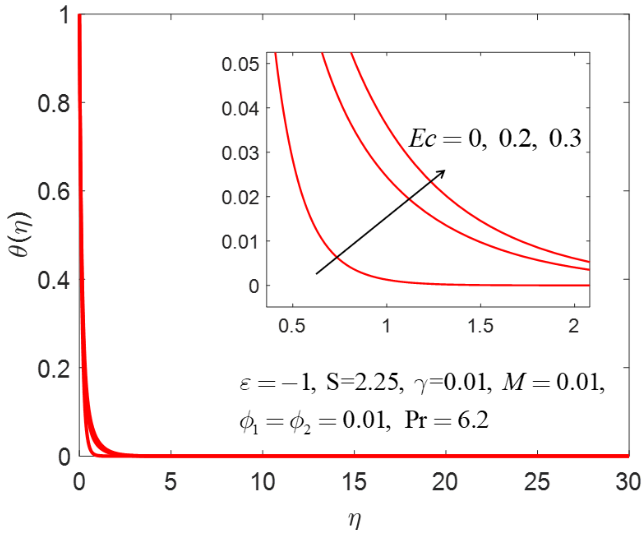

Fig. 4. $q(h)$for different values of Ec

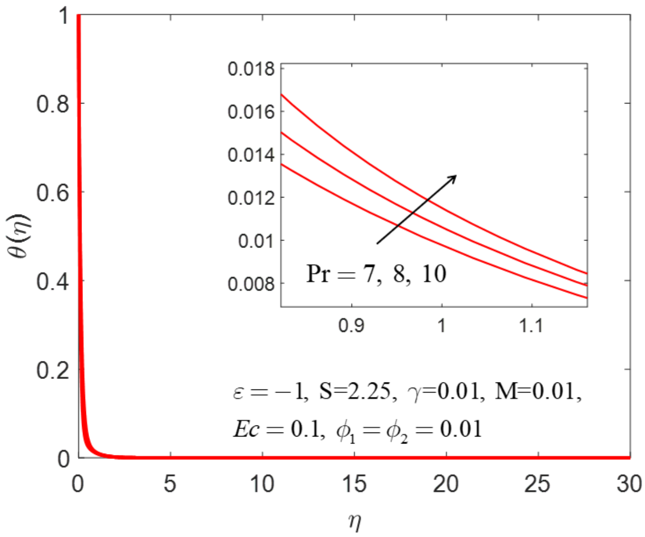

Fig. 5. $f\phi(h)$for different values of $\Pr$

Figures 6 and 7 illustrate the effects of the Williamson number (g)on velocity $f¢(h)$ and temperature $q(h)$, respectively. It is observed that as g increases, the fluid's velocity decreases while the temperature increases. Since the Williamson number represents the ratio of relaxation time to specific process time, a decrease in specific process time leads to a higher Williamson number. As a result, both velocity and boundary layer thickness are reduced. Physically, the Williamson parameter enhances the non-Newtonian behavior of the fluid by increasing its resistance due to frictional effects. Consequently, the fluid slows down, allowing more time for heat absorption from the surface, which raises the temperature.

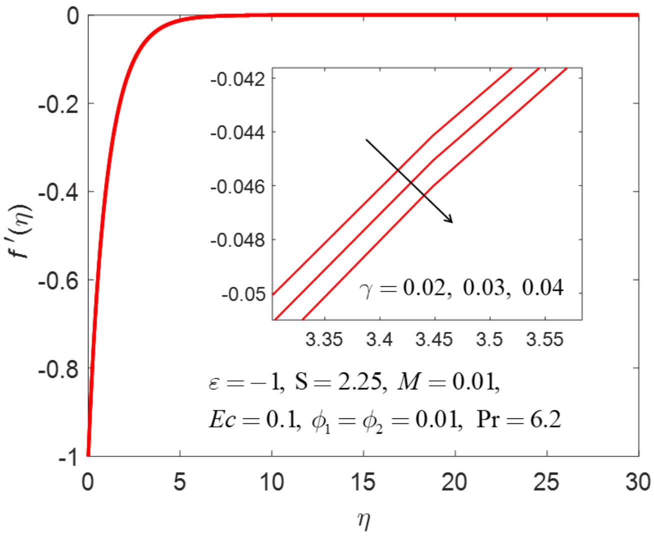

Fig. 6. $f¢(h)$for different values of g

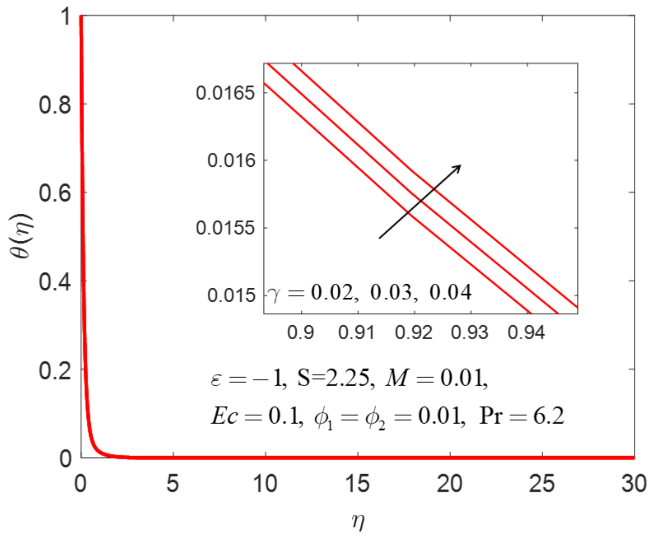

Fig. 7. $q(h)$for different values of g

Figures 8 and 9 show the effects of the suction parameter (S) on velocity $f¢(h)$ and temperature $q(h)$, respectively. It is observed that the velocity increases due to mass transfer at the suction wall, while the temperature decreases. Physically, as the suction strength in the flow increases, the velocity increases because the decelerated fluid particles are removed at the surface. As a result, heat is dissipated more quickly, leading to a decrease in the fluid temperature.

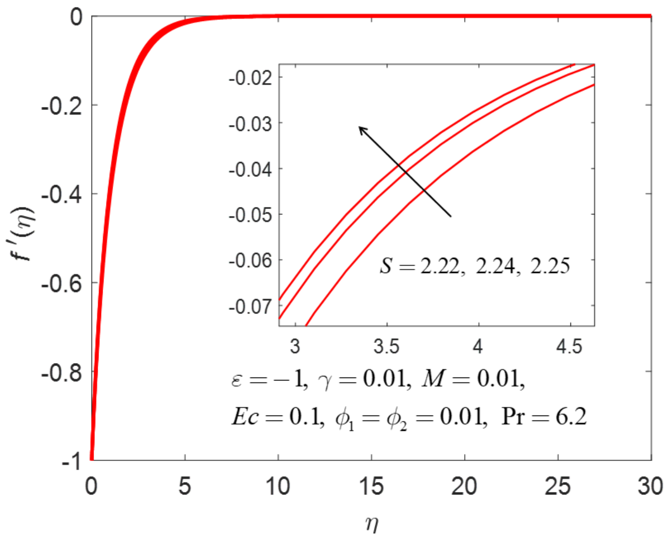

Fig. 8. $f\phi(h)$ for different values of S

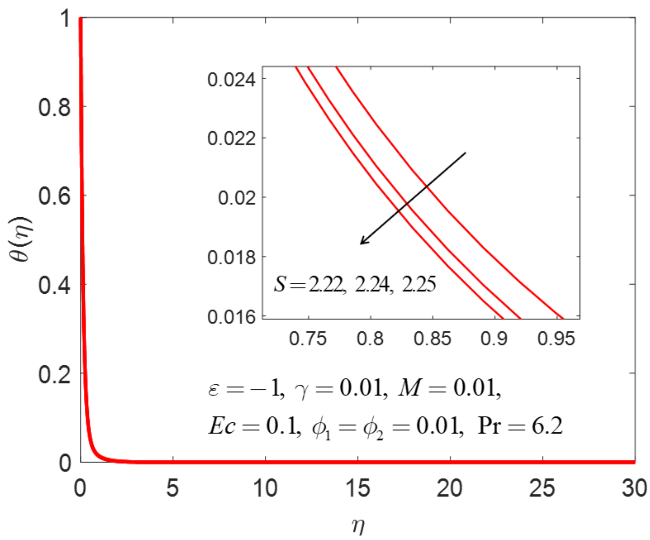

Fig. 9. $q(h)$ for different values of S

The combined effects of the Williamson parameter (g) and the volume fraction of nanoparticles (f_1, f_2) on $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ are shown in Table 6 and 7, respectively. It is observed that the values of $\mathrm{Re}_x^{1/2} C_f$ gradually decrease by 1.47% when g increases from 0 to 0.03 for the case of $f_1 = f_2 = 0.01$. However, the values of $\mathrm{Re}_x^{-1/2} Nu_x$ are only slightly affected by g, with a 0.05% decrease. For the case of $g = 0$, the values of $\mathrm{Re}_x^{1/2} C_f$ are significantly increased by 23.70% when the concentration of Cu increases from 0.5% to 2% in the base fluid. However, the values of $\mathrm{Re}_x^{-1/2} Nu_x$ slightly decreased by 0.47%.

Table 6
Values of $\mathrm{Re}_x^{1/2} C_f$ when, $e = -1, g = 0.01, M = Ec = 0, S = 2.25, \Pr = 6.2$

g	$f_1 = 0.01, = f_2 = 0.005$	$f_1 = 0.01, = f_2 = 0.01$	$f_1 = 0.01, = f_2 = 0.02$
0	0.994175254	1.085933948	1.229762975
0.01	0.985719569	1.080725376	1.225628600
0.02	0.976215875	1.075414429	1.221485537
0.03	0.964932440	1.069986681	1.217331669

Table 7
Values of $Re_x^{-1/2} Nu_x$ when, $\varepsilon = -1, \gamma = 0.01, M = Ec = 0, S = 2.25, Pr = 6.2$

γ	$\phi_1 = 0.01, \phi_2 = 0.005$	$\phi_1 = 0.01, \phi_2 = 0.01$	$\phi_1 = 0.01, \phi_2 = 0.02$
0	8.297562702	8.286588416	8.258380193
0.01	8.295658524	8.285119660	8.256933084
0.02	8.293605242	8.283651914	8.255503825
0.03	8.291286325	8.282182473	8.254091418

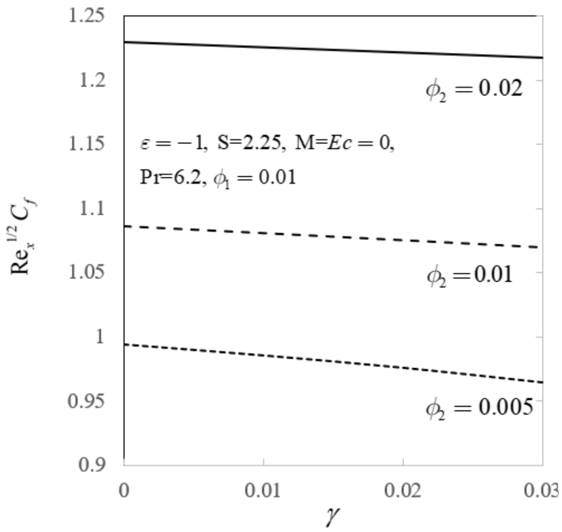

Fig. 10. Variation of $Re_x^{1/2} C_f$ against γ for different values of ϕ_2

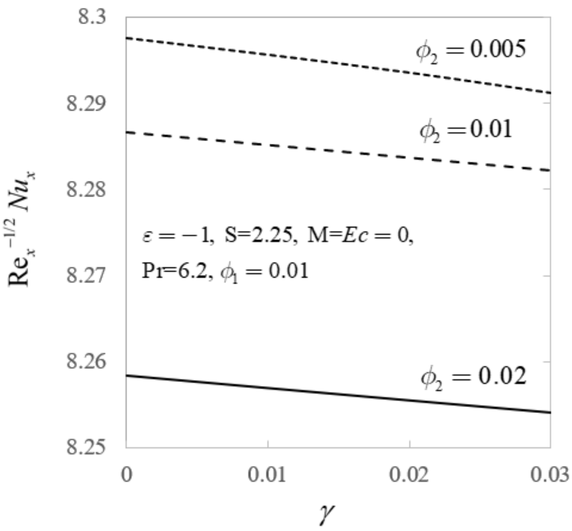

Fig. 11. Variation of $Re_x^{-1/2} Nu_x$ against γ for different values of ϕ_2

To gain more insight, Figures 10 and 11 are presented to demonstrate the simultaneous effects of the Williamson parameter (γ) and various nanoparticle concentrations (ϕ_1, ϕ_2) on $Re_x^{1/2} C_f$ and $Re_x^{-1/2} Nu_x$, respectively. The Al_2O_3-Cu/CMC-water hybrid nanofluid $(\phi_1 = 0.01, \phi_2 = 0.02)$ with the highest nanoparticle concentration exhibits the highest skin friction but the lowest Nusselt number, followed by Al_2O_3-Cu/CMC-water hybrid nanofluid $(\phi_1 = \phi_2 = 0.01)$ with a moderate concentration, and then the Al_2O_3-Cu/CMC-water hybrid nanofluid $(\phi_1 = 0.01, \phi_2 = 0.05)$ with the lowest concentration. Physically, an increase in nanoparticles concentration enhances the fluid's viscosity, causing the fluid to slow down and reducing heat transfer performance. Furthermore, an increase in the Williamson parameter (g) significantly affects both skin friction and heat transfer rate, which is consistent with its relationship to the velocity term in Eq. (7) and the energy term in Eq. (8).

4. Conclusions

This study investigates the impact of various fluid parameters, such as the magnetic parameter, Eckert number, Williamson parameter, Prandtl number, suction parameter, and nanoparticle volume fraction, on velocity and temperature profiles, as well as physical quantities like skin friction and the heat transfer rate. The results indicate that an increase in the Williamson parameter leads to a decrease in velocity but an increase in temperature. In contrast, higher values of the magnetic and suction parameters result in increased velocity but decreased temperature. It was also observed that suction allows the WHNFs molecules to gain control of the surface, enhancing the heat transfer rate. Meanwhile, a higher Williamson parameter reduces both skin friction and heat transfer rate due to the obstacles that appear by the shear-thinning phenomenon, which diminishes fluid interaction with surfaces and generates less drag force. The heat transfer rate decreases with an increase in the Eckert number and increases with a rise in the Prandtl Number in the operating fluid. Moreover, an increase in magnetic parameters and nanoparticle volume fraction results in higher skin friction but a lower heat transfer rate.

The present findings are only conclusive to the non-linear shrinking sheet. Theoretically, magnetic parameters and nanoparticles enhance the heat transfer rate. However, in this study, the higher suction strength may influence the heat transfer process. Thus, this study will be a reference for future research to further study and investigate the other physical parameters or hybrid nanomaterials that may enhance the heat transfer rate, particularly for the shrinking sheet case.

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