



Optimizing the Operating Modes of Axial Pumping Stations Using Mathematical Modeling and Control Algorithms for Sustainable Water and Energy Management

Mamatkobil Esonturdiyev^{1,*}, Aybek Seytov¹, Roziya Haydarova², Mexribon Solayeva¹, Isroilbek Ergashev³, Iqbol Qaynarova³, Khayriddin Rakhmonov⁴, Lola Kuzibaevna Narimbaeva³, Toshtemirova Dilbar Bozarovna⁵

- ¹ The University of World Economy and Diplomacy, Tashkent, Uzbekistan
² Termiz State University, Termiz, Uzbekistan
³ Chirchik State Pedagogical University, Chirchik, Uzbekistan
⁴ Bukhara State Technical University, Bukhara, Uzbekistan
⁵ Academic Lyceum named after M.S. Vosiqova under Tashkent State University of Law, Uzbekistan

ARTICLE INFO

Article history:

Received 13 May 2026
Received in revised form 27 July 2026
Accepted 7 August 2026
Available online 5 October 2026

Keywords:

Axial pump; Control algorithm; Energy efficiency;
Irrigation system; Mathematical modeling;
Pumping station; Sustainable water management

ABSTRACT

Efficient operation of axial pumping stations requires coordinated management of hydraulic conditions, pump-unit combinations, water delivery, and energy consumption. This study develops a mathematical modeling and control approach for optimizing the operating modes of axial pumping stations in irrigation systems. Hydraulic parameters, pump characteristics, operational constraints, and energy requirements are integrated into an optimization framework to determine advantageous flow distributions among multiple pumping units. Analytical optimization, including the Lagrange approach, is combined with computational analysis using operational data from the irrigation system. The results identify feasible operating modes under different hydraulic conditions and demonstrate the importance of coordinated pump operation for controlling water flow and energy use. The proposed approach provides an engineering framework for improving pumping-station operation while maintaining required hydraulic conditions. The findings are relevant to sustainable water and energy management and support resource-efficient operation of irrigation infrastructure.

1. Introduction

Pumping stations are essential components of irrigation and water-management infrastructure because they regulate water delivery across hydraulic networks under changing demand and operating conditions. Their operation, however, is energy intensive, and inefficient selection or

* Corresponding author.
E-mail address: esonturdiyev80@mail.ru

coordination of pumping units can increase electricity consumption while reducing hydraulic performance. Consequently, optimization of pumping-station operation requires simultaneous consideration of water demand, hydraulic constraints, pump characteristics, and energy requirements. Mathematical optimization provides an effective framework for addressing these interacting variables and identifying operating conditions that satisfy hydraulic requirements while minimizing energy use [1].

For multi-pump stations, optimization must account for the combined operating state of active units rather than the performance of individual pumps alone. Mathematical programming and related optimization approaches have therefore been widely applied to pump selection, flow allocation, and coordinated station operation [2, 3]. However, most such approaches formulate the optimization within specified hydraulic conditions, whereas irrigation canals may impose operating constraints that vary continuously with water level and discharge.

Energy consumption is another critical consideration in pumping-system management. Pump scheduling determines when individual units operate and how water demand is distributed among them, directly influencing energy use and operating costs. Optimization-based scheduling can improve energy efficiency by coordinating pumping decisions with hydraulic system requirements [4]. Similarly, optimal-control approaches allow pumping-station operation to be adjusted according to changing system conditions, providing a systematic means of improving energy performance while satisfying operational constraints [5]. These studies establish the importance of integrating hydraulic behavior and energy management within a unified optimization framework.

Computational approaches have further expanded the possibilities for pumping-system optimization. Mathematical models can represent nonlinear relationships among flow rate, hydraulic head, energy consumption, and operational constraints, while optimization algorithms identify feasible operating states from a large number of possible pump combinations. Energy-oriented scheduling models have demonstrated that pump operation can be formulated as an optimization problem involving hydraulic constraints and energy requirements [6]. Such approaches are particularly relevant to large irrigation systems, where variations in canal water level and flow demand can continuously change the permissible and advantageous operating modes of pumping units.

Despite these developments, an important operational challenge remains in irrigation pumping systems where feasible pump combinations change directly with canal water level and hydraulic state. In axial pumping stations, a pump combination that is technically and energetically acceptable at one water level may become infeasible as the canal level decreases because of changing discharge conditions, hydraulic losses, equipment limitations, and potential cavitation. Therefore, optimization should not only determine an energy-efficient pump combination but also continuously redefine the feasible operating space according to the hydraulic state of the connected canal. This limitation is particularly important for axial pumping stations because their permissible operating modes are strongly dependent on water level, discharge, hydraulic losses, and cavitation-related restrictions. In the investigated Jizzakh cascade, decreasing canal water level progressively reduces the number of permissible pump-unit combinations, meaning that the optimization domain itself changes with the hydraulic state. This water-level-dependent feasible operating region forms a central constraint in the present optimization framework. In the system investigated in the present study, the hydraulic characteristics of the South-Golodnostepsky main canal and the operating conditions of the Jizzakh cascade of pumping stations provide the boundary conditions for determining feasible pump-unit combinations. The analysis considers canal geometry, hydraulic parameters, water-level-dependent operating modes, and time-dependent flow behavior required for system-level evaluation.

Therefore, this study aims to develop a hydraulic-state-dependent optimization framework for axial pumping stations in which pump-unit combinations are selected according to both energy performance and dynamically changing canal conditions. The approach integrates pump characteristics and energy consumption with canal geometry, hydraulic losses, water-level-dependent operating constraints, and cavitation-related feasibility limits to identify technically admissible and energetically advantageous operating modes.. The principal contribution of this study is the incorporation of water-level-dependent hydraulic feasibility into the optimization of axial pumping-station operation. Unlike conventional pump-scheduling approaches that primarily optimize pump combinations under predefined hydraulic conditions, the proposed framework restricts candidate operating modes according to canal water level, hydraulic losses, permissible pump-unit combinations, and potential cavitation conditions. The framework further considers the time-dependent hydraulic response of the connected open-channel system, allowing pump-operation decisions to be evaluated within dynamically changing canal conditions. The analysis additionally considers the relationship among water-resource management, pumping-station control, and electricity-consumption management, which are integral components of the investigated system.

From a sustainability perspective, improving pumping-station efficiency is relevant to the Sustainable Development Goals (SDGs) because irrigation infrastructure simultaneously involves water and energy resources. Optimized operation that maintains required water delivery while reducing unnecessary energy use can contribute to more resource-efficient water management. Accordingly, the proposed engineering framework is particularly relevant to SDG 6 (Clean Water and Sanitation) through efficient water management and SDG 7 (Affordable and Clean Energy) through improved energy-use efficiency in pumping operations.

2. Literature Review

2.1 Mathematical Modeling and Optimization of Pumping Stations

Pumping-station optimization is a system-level problem involving hydraulic performance, pump characteristics, operating constraints, and energy consumption. In multi-pump installations, the required discharge can be supplied through different combinations of operating units; therefore, identifying an appropriate combination is essential for minimizing energy use while maintaining hydraulic requirements. Optimization models provide a systematic means of evaluating these alternatives and selecting operating states according to defined objective functions and constraints [2, 3].

Pump scheduling extends this concept by determining how pumping units should operate under changing hydraulic and demand conditions. Optimization techniques such as nonlinear programming, mixed-integer formulations, and branch-and-bound approaches have been developed to identify energy- and cost-efficient operating schedules [6, 7]. These methods are particularly useful for systems involving multiple feasible pump combinations because manual or individual-unit control may not identify the global or system-level optimum.

Recent developments have further incorporated advanced computational algorithms into pumping-system management. Optimal-control and pump-scheduling approaches have been developed to coordinate pump operation under hydraulic and energy constraints [8, 9]. These developments demonstrate substantial progress in coordinated pump scheduling and system-level optimization. However, comparatively less attention has been given to optimization frameworks in which the feasible pump combinations themselves change with open-channel water level, hydraulic

losses, and cavitation-related limits. This distinction is particularly important for axial pumping stations connected directly to irrigation canals..

2.2 Hydraulic Conditions and Pump Operating Modes

Hydraulic conditions not only influence pump efficiency but can also redefine the feasible operating range of axial pumping units. Variations in canal water level, discharge, hydraulic losses, and system resistance can progressively eliminate otherwise attractive pump combinations from the feasible solution space. Therefore, these hydraulic variables should be treated as active optimization constraints rather than as fixed external inputs.. Variations in water level, flow rate, hydraulic losses, and system resistance modify the operating point and may restrict the number or combination of pumps that can operate safely. Consequently, hydraulic parameters should be incorporated directly into the optimization process rather than treated independently from pump scheduling.

For irrigation pumping stations, this relationship is particularly important because canal conditions vary spatially and temporally. Mathematical models can integrate canal geometry, flow conditions, and pumping requirements to determine feasible operating states. Dynamic scheduling approaches further allow pumping decisions to respond to variations in system conditions, thereby improving coordination between water delivery and energy use [10].

The investigated system illustrates this dependence clearly. The South-Golodnostepsky main canal is characterized by defined geometric and hydraulic parameters, while the permissible operating combinations of the Jizzakh main pumping station change with the water level at the UVPK-400 control section. At relatively high water levels, a broader range of pump-unit combinations is available, whereas decreasing water levels progressively restrict feasible operation and may eventually lead to potential cavitation conditions. This relationship provides the hydraulic basis for incorporating operating constraints into the optimization model.

2.3 Energy-Efficient Control and Sustainable Operation

Energy efficiency is a central objective of pumping-station optimization because water transport requires substantial electrical input. Coordinated control can reduce unnecessary energy consumption by allocating discharge among pumping units according to their operating characteristics rather than simply activating units independently. Optimal-control strategies therefore connect hydraulic requirements with electricity-consumption management [5].

Modern approaches increasingly integrate hydraulic efficiency, system-level analysis, and mathematical programming into pumping-system optimization [10, 11]. Although these approaches differ computationally, they share the principle that pumping stations should be managed as integrated hydraulic-energy systems. The remaining challenge is to embed dynamically changing canal conditions within this integrated optimization framework.

This concept is directly relevant to the Jizzakh cascade, where water-resource management and electricity-consumption management are interconnected. The system representation includes central control, individual pumping stations, hydraulic structures, and electricity-management components, emphasizing the need for coordinated hydraulic and energy decisions. Accordingly, mathematical optimization provides an engineering basis for selecting operating modes that satisfy water-delivery requirements while limiting inefficient energy use.

3. Methodology

3.1 Research Framework

This study employed mathematical modeling and optimization to determine advantageous operating modes of axial pumping stations. The analysis integrated hydraulic parameters, pump characteristics, energy consumption, and operational constraints within a system-level framework. Analytical optimization, including the Lagrange approach, was used to determine the distribution of flow among operating pumping units. Computational analysis was then applied to operational data from the irrigation system to evaluate feasible operating conditions. Hydraulic and equipment parameters were incorporated into a functional minimization model based on energy consumption. Hydraulic and equipment parameters were incorporated into a functional minimization problem based on energy consumption.

The optimization problem can be represented conceptually as minimizing the total energy-related objective function ($\min F(Q_1, Q_2, \dots, Q_n)$). Then, subject to the required total discharge is $\sum_{i=1}^n Q_i = Q_{\text{req}}$, and the hydraulic and operational constraints of each pumping unit. Here, Q_i represents the discharge assigned to the i -th pumping unit, while Q_{req} represents the required system discharge. The Lagrange approach provides a mathematical basis for distributing the required flow among operating units while satisfying the total-flow constraint.

3.2 Hydraulic System and Boundary Conditions

The South-Golodnostepsky main canal and the Jizzakh cascade of pumping stations were used as the investigated hydraulic system. The canal parameters included channel width (b), side-slope coefficient (m), roughness coefficient (n), flow depth (y), longitudinal slope (i), and canal length (l). These parameters define the hydraulic boundary conditions required for evaluating flow behavior and pump operation. The dataset provides $b = 19$ meters, $m = 8$, $n = 0.02$, $y = 0.17$ meters, $i = 0.00006$, and $l = 25,500$ meters. These values were used as the hydraulic input parameters for the optimization analysis. The operating modes were additionally constrained by water level at the UVPK-400 control section. The available combinations of pumping units decrease as water level declines, and operation near approximately 3.6 m and below approaches a potential cavitation region according to the analysis. The optimization domain was explicitly restricted by water-level-dependent hydraulic and mechanical feasibility. Candidate pump combinations that violated the permissible hydraulic range or approached the identified cavitation region were excluded before energy-based comparison. Consequently, the optimization seeks the lowest-energy solution only within the hydraulically admissible operating space.

3.3 Determination of Optimal Pump-Unit Operation

The optimization procedure evaluated the relationship between pumping-station flow rate and relative changes in unit consumption. Optimal flow distribution was identified from the characteristic relationships among operating units. For pumping stations containing different types of units, graphical intersections of the corresponding loss characteristics were used to identify advantageous distributions of water flow.

The procedure can be summarized as: (i) defining the required hydraulic operating condition; (ii) identifying feasible combinations of pumping units; (iii) distributing the required discharge among active units; (iv) evaluating the associated energy-related objective function; and (v) selecting the feasible operating mode providing the most advantageous system performance.

The distinctive feature of the procedure is that the feasible set of pump combinations is updated according to the prevailing hydraulic condition before energy-based selection is performed. The resulting operating state must simultaneously satisfy discharge requirements, hydraulic conditions, equipment limitations, and energy considerations.

3.4 Water and Electricity Management Framework

The optimization was embedded within the broader management structure of the Jizzakh cascade of pumping stations. Water-resource management involved interaction among the central control system, pumping stations, and hydraulic structures. Electricity-consumption management was considered together with pumping-station operation because changes in the number and operating mode of active units directly influence energy demand. These two system structures are subsequently presented.

Time-dependent variations in water flow along sections of the South-Golodnostepsky main canal were also considered for operational planning. The calculations demonstrate that operating time changes with flow conditions and adjustments, providing information required for real-time control and management.

4. Results and Discussion

4.1 Optimal Flow Distribution among Pumping Units

The relationship between pumping-station flow rate and the relative increase in unit consumption is presented in Figure 1. The characteristic curves represent different pumping units and provide a graphical basis for determining how the required discharge should be distributed. The intersection of the characteristic relationships identifies operating conditions at which the units can share the required flow with more favorable energy performance.

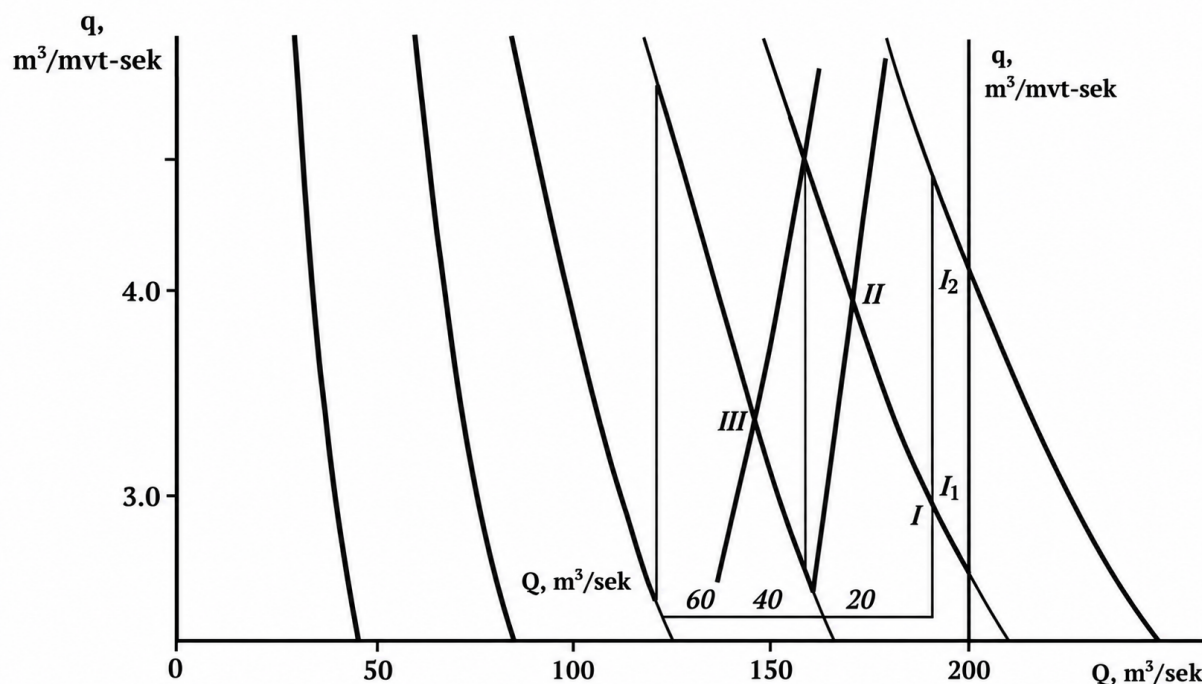


Fig. 1. Characteristics of relative increases in unit consumption depending on the pumping-station flow rate

required water discharge must be distributed among available pumps. Second, the most advantageous allocation depends on the characteristics of the active units. Therefore, optimization algorithms provide a rational basis for selecting operating modes when several configurations can satisfy the same overall water demand.

4.2 Integrated Water-Resource and Electricity-Consumption Management

Optimization of individual pumping units is only one component of efficient station operation. The investigated Jizzakh cascade forms part of a larger hydraulic network in which pumping stations, canals, control structures, water-management units, and electricity-management systems interact. Figure 3 illustrates the organizational structure used for water-resource management and regulation of the operating modes of the Jizzakh cascade.

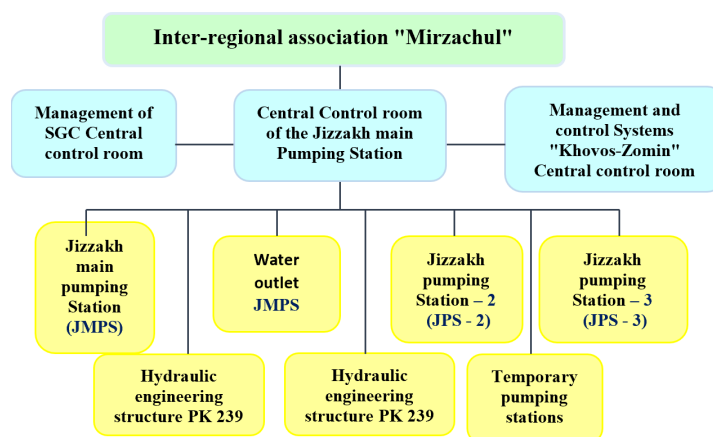


Fig. 3. Structure of water-resource management and operating modes of the Jizzakh cascade of pumping stations

The structure demonstrates that pumping-station operation is linked to centralized management and hydraulic information from different parts of the system. This arrangement is important because a control decision taken at one pumping station may influence downstream water availability and the operating requirements of other stations. Coordinated operation therefore allows water-distribution decisions to be based on system conditions rather than isolated local observations.

Such integration is consistent with the increasing use of dynamic scheduling and system-state information in pump optimization. Modern control frameworks employ continuously updated hydraulic information to determine pump combinations that satisfy changing demands while maintaining efficient operation [10, 11]. Although the present study uses a mathematical and algorithm-based framework rather than an advanced digital-twin architecture, the underlying engineering principle is similar: operating decisions should respond to the state of the entire hydraulic system.

Energy management is represented in Figure 4, which shows the structure used for electricity-consumption management within the Jizzakh cascade. The figure demonstrates the direct relationship between electricity supply and pumping-station operation. Changes in the number of active units, their flow distribution, and their operating duration influence the total electrical demand of the system. Consequently, water-resource management and electricity-consumption management cannot be optimized independently.

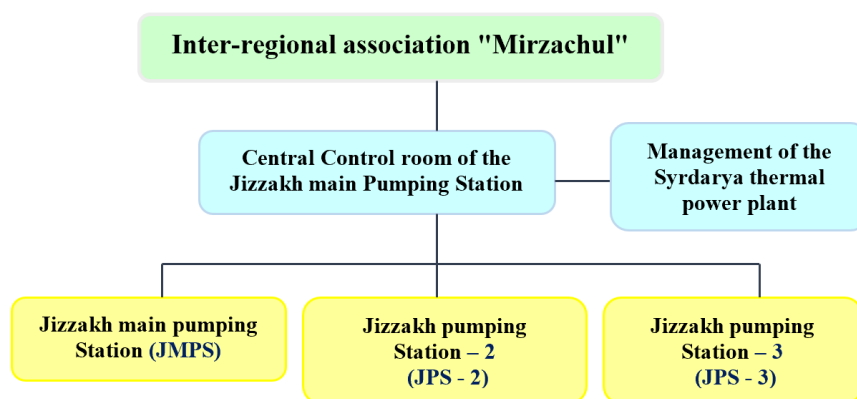


Fig. 4. Structure of electricity-consumption management of the Jizzakh cascade of pumping stations

This finding is particularly important for large irrigation pumping systems because achieving the required water delivery does not automatically imply efficient energy use. Two operating configurations can potentially supply a similar total discharge but require different energy inputs. Pump scheduling and optimal-control studies have similarly shown that coordination between hydraulic and energy variables can substantially influence operating efficiency [4, 5]. Thus, the management structures in Figures 3 and 4 provide the operational framework within which the mathematical optimization results should be implemented.

4.3 Hydraulic Parameters and Canal Operating Conditions

The hydraulic behavior of the pumping system is influenced by the characteristics of the South-Golodnostepsky main canal. The dataset provides a channel width ($b = 19$ meters), side-slope coefficient ($m = 8$), roughness coefficient ($n = 0.02$), flow depth ($y = 0.17$ meters), longitudinal slope ($i = 0.00006$), and total canal length ($l = 25,500$ meters). These parameters define the hydraulic environment in which the pumping stations operate and constitute essential boundary conditions for modeling flow behavior.

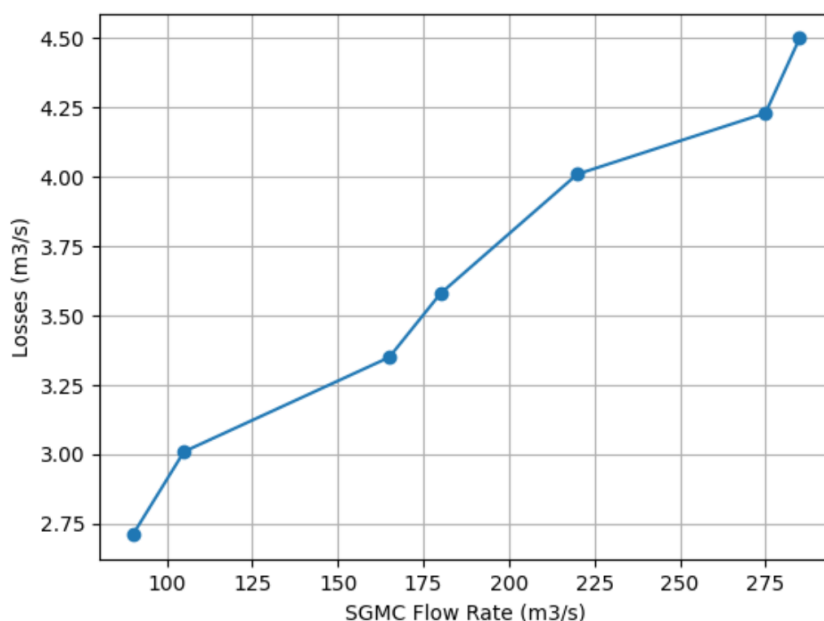


Fig. 5. Relationship between SGMC flow rate and calculated losses for the channel section between PK-145 and PK-400

The calculated operating behavior of the canal section between PK-145 and PK-400 is shown in Figure 5. The calculated curves demonstrate that canal operation changes with water level and discharge. This relationship is critical because pumping-station control must respond to hydraulic conditions within the connected canal rather than treating the required discharge as an independent fixed parameter. Numerical modeling of open-channel water-resource problems provides an appropriate basis for representing these relationships and evaluating the consequences of changing hydraulic conditions [13].

The importance of hydraulic state becomes particularly clear when the permissible operating modes of the Jizzakh main pumping station are examined. At water levels of approximately 6.4-5.6 m, the system permits a relatively broad range of operating configurations, including combinations involving one to three units. As the water level decreases toward approximately 5 m and below, the available combinations become progressively restricted. At still lower levels, around 4.4-3.6 m, operation becomes increasingly constrained, and near 3.6 m and below, the analysis identifies a potential cavitation region.

These findings reveal the key distinction between unconstrained pump-energy optimization and the present hydraulic-state-dependent framework. The mathematically lowest-energy combination cannot necessarily be selected because decreasing canal water level progressively changes the feasible operating domain. Hydraulic safety therefore defines the admissible pump combinations first, after which energy minimization is performed within that restricted domain. An operating configuration must first satisfy hydraulic and mechanical feasibility requirements. Therefore, optimization is subject to a hierarchy of constraints: safe hydraulic operation defines the feasible solution space, and energy minimization is subsequently performed within that space.

The result also explains why static pump scheduling may be insufficient for irrigation systems with changing canal levels. A combination that is permissible at 6.0 m may no longer be acceptable after the water level declines. Consequently, control algorithms should be capable of updating permissible pump combinations as hydraulic conditions change. Previous studies similarly emphasize that hydraulic and energy-efficiency considerations should be evaluated together when optimizing water-supply systems [12].

Open-channel modeling is also important for determining how changes propagate through the irrigation system. Finite-element and computational approaches have been applied to unsteady water movement in open channels and can provide additional tools for representing spatial and temporal hydraulic variation [14]. In the present case, hydraulic modeling provides the connection between canal conditions and pump-operation decisions.

4.4 Time-Dependent Flow Behavior and Operational Control

The temporal response of the canal system is presented in Figure 6, which shows calculated changes in operating time for different sections and flow conditions. The calculated results demonstrate that water movement and system response are time dependent. Changes in discharge do not instantaneously produce uniform conditions throughout the canal; therefore, the timing of operational adjustments must be considered when controlling pumping stations. This factor is especially relevant for cascaded systems, where a change in upstream pumping operation may alter downstream hydraulic conditions after a finite travel or response time.

The time-dependent calculations provide a second layer of operational constraint because changes in canal discharge require a finite hydraulic response time. Pump switching and discharge adjustments should therefore be coordinated not only with the instantaneous hydraulic state but also with the expected propagation of flow changes through the canal system. Pump-unit selection,

switching, and discharge adjustment should be coordinated with the expected hydraulic response of the canal. Excessively rapid or poorly coordinated changes may produce temporary mismatches between pumping capacity and actual system requirements. Conversely, incorporating system-response time into the control algorithm allows operational changes to be scheduled more consistently with hydraulic conditions.

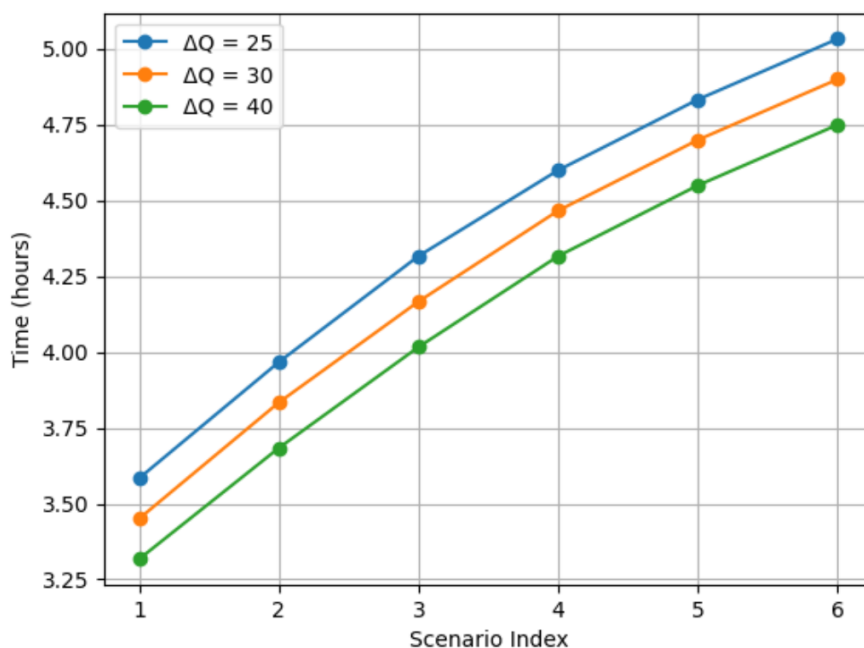


Fig. 6. Calculated time variation for different flow-change scenarios ($\Delta Q = 25, 30$, and 40) in sections of the South-Golodnostepsky main canal

The time-dependent hydraulic response is particularly important in cascade pumping systems connected through open channels. Previous studies have demonstrated that hydrodynamic modeling can be coupled with pumping-station optimization to account for interactions among discharge, hydraulic head, and canal operating conditions [15]. Changes in pumping discharge also require a finite period to propagate through the canal system, meaning that operational adjustments should consider hydraulic response time rather than assuming instantaneous changes [16]. More recent real-time scheduling approaches have further incorporated dynamically updated hydraulic information into cascade pumping-station control under changing water-demand and canal conditions [17]. These findings support the present consideration of both the current hydraulic state and the expected temporal response of the canal when determining feasible operating modes.

The combination of Figures 5 and 6 therefore provides two complementary pieces of information. Figure 5 describes permissible operating behavior as hydraulic conditions vary, whereas Figure 6 shows that transitions between conditions occur over time. Together, these results support control strategies in which pump selection is updated according to both the current hydraulic state and the expected temporal evolution of the canal system.

4.5 Engineering Significance of the Optimization Framework

The collective results demonstrate that efficient operation of axial pumping stations requires integration of pump characteristics, hydraulic boundary conditions, flow-distribution requirements, temporal behavior, and electricity consumption. Each component is represented within the results:

Figures 1 and 2 describe optimal distribution among pumping units; Figures 3 and 4 show the coordination of water and electricity management; and Figures 5 and 6 describe hydraulic operating conditions and their temporal variation.

The engineering significance of the framework lies in its treatment of hydraulic feasibility as a dynamic constraint on optimization. Canal water level, permissible pump combinations, hydraulic losses, and potential cavitation conditions define a changing feasible operating region within which flow allocation and energy performance are evaluated. The results show that operating decisions are constrained not only by energy consumption but also by canal water level, permissible pump combinations, hydraulic losses, and potential cavitation conditions. The control problem is therefore multi-variable and constrained, requiring mathematical optimization rather than simple selection of the nominally most efficient individual pump.

The present analysis is also consistent with previous work by the authors on applying mathematical modeling and optimization to pumping stations in irrigation systems. In the current framework, the engineering interpretation is strengthened by explicitly connecting optimal flow distribution with water-resource management, electricity-consumption management, canal operating modes, and time-dependent hydraulic behavior.

From an operational viewpoint, permissible pump combinations should be updated as canal water levels and hydraulic conditions change, and energy minimization should subsequently be performed only among those hydraulically feasible alternatives. This sequence prevents an energetically attractive but hydraulically unsafe operating mode from being selected.

The results also support the development of automated or semi-automated control algorithms. Algorithm-based planning has also been applied to water management in irrigation systems, demonstrating that coordinated operational decisions can improve the allocation and management of water resources under varying system conditions [18]. Once hydraulic variables, equipment characteristics, and constraints are incorporated into the mathematical model, candidate operating modes can be evaluated computationally. This provides a basis for decision support in which operators select among feasible modes according to current water and energy requirements. Further development could incorporate more advanced real-time data acquisition and adaptive optimization techniques, but such extensions would require additional operational data beyond those available in the present study.

4.6. Implications for Sustainable Water and Energy Management

The sustainability contribution of the proposed approach arises from the close relationship between water delivery and electricity consumption. Irrigation pumping stations consume energy specifically to transport and regulate water; therefore, inefficient operation can simultaneously increase water-management inefficiencies and unnecessary energy consumption. Mathematical optimization provides a systematic means of reducing avoidable energy use while maintaining the required hydraulic conditions and water-delivery targets.

The results are particularly relevant to SDG 6 because efficient operation of irrigation pumping infrastructure supports more effective allocation, conveyance, and delivery of water resources. Previous studies on irrigation and water-supply systems have shown that pumping operation should be coordinated with water demand, hydraulic conditions, and system constraints to improve water-management efficiency [19]. In addition, energy-efficient pump operation can reduce unnecessary electricity consumption while maintaining the required water-delivery performance, thereby strengthening the connection between sustainable water management and pumping-system optimization [20]. In the present study, the contribution to SDG 6 is therefore associated primarily

with improving the operational efficiency of water conveyance and distribution rather than with water-quality treatment or remediation.

The proposed optimization framework is also relevant to SDG 7 (Affordable and Clean Energy), since the selection of appropriate pump-unit combinations and operating schedules can reduce avoidable electricity consumption. In pumping systems, water and energy performance are inherently interconnected because any unnecessary hydraulic resistance, inappropriate pump selection, or inefficient operating mode can increase the energy required to deliver a specified flow rate. Therefore, improving the hydraulic operating mode can contribute simultaneously to more efficient water distribution and more rational energy utilization. More broadly, sustainable engineering studies have emphasized the importance of improving energy-resource efficiency and developing alternative strategies for more sustainable energy utilization [21].

Such an approach is relevant to the present pumping analysis because hydraulic performance cannot be interpreted solely from numerical outputs; the relationships among flow rate, head, operating conditions, and energy consumption must also be understood physically. However, the sustainability interpretation should remain proportional to the available evidence. The present analysis demonstrates optimized hydraulic operating modes and provides an engineering framework for reducing water- and energy-related inefficiencies, but it does not directly quantify life-cycle environmental impacts, carbon-emission reductions, long-term regional water savings, or changes in water quality. These broader sustainability effects would require additional field measurements, electricity-consumption records, environmental assessments, and long-term operational datasets.

Mathematical modeling and control algorithms therefore enable the Jizzakh pumping system to be interpreted as an integrated hydraulic-energy system. The principal engineering contribution is not simply the identification of a single optimal pump combination, but the establishment of a framework in which operating modes can be selected according to changing water levels, hydraulic constraints, required flow rates, and energy considerations. This system-level perspective provides a practical basis for improving the reliability, efficiency, and resource-conscious operation of axial pumping stations.

5. Conclusion

This study developed a mathematical optimization framework for selecting operating modes of axial pumping stations under changing hydraulic conditions. The results demonstrate that efficient operation depends on coordinated flow distribution among pumping units, canal water level, hydraulic constraints, time-dependent flow behavior, and electricity-consumption management. The analysis identified progressively restricted operating modes as water levels decreased and highlighted potential cavitation conditions at low levels. The main contribution of the study is the formulation of a water-level-dependent feasible operating domain for axial pumping stations, in which hydraulic losses, permissible pump combinations, and potential cavitation constraints are evaluated before energy-based optimization. The results further show that canal response time should be considered when operational adjustments are implemented. This framework provides a practical basis for hydraulic-state-dependent control of axial pumping stations under changing irrigation-canal conditions. The approach supports more reliable irrigation-water management and contributes to resource-efficient operation relevant to SDGs 6 and 7.

References

- [1] Brás, M., A. Moura, and A. Andrade-Campos. "Cost Efficiency in Water Supply Systems: An Applied Review on Optimization Models for the Pump Scheduling Problem." *European Journal of Operational Research* 323, no. 1 (2025): 1–19. <https://doi.org/10.1016/j.ejor.2024.07.039>.
- [2] Moradi-Jalal, M., M. A. Mariño, and A. Afshar. "Optimal Design and Operation of Irrigation Pumping Stations." *Journal of Irrigation and Drainage Engineering* 129, no. 3 (2003): 149–154. [https://doi.org/10.1061/\(ASCE\)0733-9437\(2003\)129:3\(149\)](https://doi.org/10.1061/(ASCE)0733-9437(2003)129:3(149)).
- [3] Moradi-Jalal, M., and B. W. Karney. "Optimal Design and Operation of Irrigation Pumping Stations Using Mathematical Programming and Genetic Algorithm (GA)." *Journal of Hydraulic Research* 46, no. 2 (2008): 237–246. <https://doi.org/10.1080/00221686.2008.9521858>.
- [4] Luna, T., J. Ribau, D. Figueiredo, and R. Alves. "Improving Energy Efficiency in Water Supply Systems with Pump Scheduling Optimization." *Journal of Cleaner Production* 213 (2019): 342–356. <https://doi.org/10.1016/j.jclepro.2018.12.190>.
- [5] Tang, Y., G. Zheng, and S. Zhang. "Optimal Control Approaches of Pumping Stations to Achieve Energy Efficiency and Load Shifting." *International Journal of Electrical Power & Energy Systems* 55 (2014): 572–580. <https://doi.org/10.1016/j.ijepes.2013.10.023>.
- [6] Shi, H., and F. You. "Energy Optimization of Water Supply System Scheduling: Novel MINLP Model and Efficient Global Optimization Algorithm." *AIChE Journal* 62, no. 12 (2016): 4277–4296. <https://doi.org/10.1002/aic.15332>.
- [7] Menke, R., E. Abraham, P. Parpas, and I. Stoianov. "Exploring Optimal Pump Scheduling in Water Distribution Networks with Branch and Bound Methods." *Water Resources Management* 30 (2016): 5333–5349. <https://doi.org/10.1007/s11269-016-1490-8>.
- [8] Ormsbee, L. E., and K. E. Lansey. "Optimal Control of Water Supply Pumping Systems." *Journal of Water Resources Planning and Management* 120, no. 2 (1994): 237–252. [https://doi.org/10.1061/\(ASCE\)0733-9496\(1994\)120:2\(237\)](https://doi.org/10.1061/(ASCE)0733-9496(1994)120:2(237)).
- [9] McCormick, G., and R. S. Powell. "Optimal Pump Scheduling in Water Supply Systems with Maximum Demand Charges." *Journal of Water Resources Planning and Management* 129, no. 5 (2003): 372–379. [https://doi.org/10.1061/\(ASCE\)0733-9496\(2003\)129:5\(372\)](https://doi.org/10.1061/(ASCE)0733-9496(2003)129:5(372)).
- [10] Coelho, B., and A. Andrade-Campos. "Efficiency Achievement in Water Supply Systems—A Review." *Renewable and Sustainable Energy Reviews* 30 (2014): 59–84. <https://doi.org/10.1016/j.rser.2013.09.010>.
- [11] D'Ambrosio, C., A. Lodi, S. Wiese, and C. Bragalli. "Mathematical Programming Techniques in Water Network Optimization." *European Journal of Operational Research* 243, no. 3 (2015): 774–788. <https://doi.org/10.1016/j.ejor.2014.12.039>.
- [12] Vilanova, M. R. N., and J. A. P. Balestieri. "Energy and Hydraulic Efficiency in Conventional Water Supply Systems." *Renewable and Sustainable Energy Reviews* 30 (2014): 701–714. <https://doi.org/10.1016/j.rser.2013.11.024>.
- [13] Seytov, A., O. Abduraxmonov, D. Anvarov, M. Esonturdiyev, and A. Jumabaev. "Numerical Implementation of Optimal Water Resource Management Problems in Open Channels Using Python." *Uzbek Mathematical Journal* 69, no. 1 (2025): 133–140. <https://doi.org/10.29229/uzmj.2025-1-13>.
- [14] Seytov, A., L. Varlamova, A. Khudayberdiev, N. Uteuliev, S. Yadgarov, and D. Seytimbetov. "Usage of Finite Element Method for Modeling Two-Dimensional Unsteady Water Movement in Open Channels." *AIP Conference Proceedings* 3147, no. 1 (2024): 030034. <https://doi.org/10.1063/5.0210332>.
- [15] Zhang, Z., X. Lei, Y. Tian, L. Wang, H. Wang, and K. Su. "Optimized Scheduling of Cascade Pumping Stations in Open-Channel Water Transfer Systems Based on Station Skipping." *Journal of Water Resources Planning and Management* 145, no. 7 (2019): 05019011.
- [16] Zhang, N., H. Ren, and F. Lin. "Study on the Hydraulic Response of an Open-Channel Water Transmission Project after Flow Switching." *Water* 15, no. 18 (2023): 3201.
- [17] Xia, H., C. Wang, J. Sun, X. Lei, and H. Wang. "Multi-Scale Closed-Loop Coupled Real-Time Water Quantity Optimization Scheduling of Cascade Pumping Station in Water Supply Canal Systems." *Journal of Hydrology* 641 (2024): 131802.
- [18] Turaev, R., A. Seytov, B. Hollozov, R. Haydarova, and S. Kuldasheva. "Algorithms for Planning Water Management in Irrigation Systems." *AIP Conference Proceedings* 3045, no. 1 (2024): 040040. <https://doi.org/10.1063/5.0197799>.
- [19] González Perea, R., I. Fernández García, M. Martín Arroyo, J. A. Rodríguez Díaz, and E. Camacho Poyato. "Optimal Pumping Flow Algorithm to Improve Pumping Station Operations in Irrigation Systems." *Agriculture* 14, no. 3 (2024): 463.
- [20] Naval, N., and J. M. Yusta. "Optimal Short-Term Water-Energy Dispatch for Pumping Stations with Grid-Connected Photovoltaic Self-Generation." *Journal of Cleaner Production* 316 (2021): 128386.

- [21] Umar, A., A. A. Gbenga, M.-N. O. Yusuf, A. Faridah, A. T. Emmanuel, and A. F. Adekitan. "Production and Characterization of Briquettes from Agricultural Wastes for Sustainable Energy Solutions." *ASEAN Journal for Science and Engineering in Materials* 4, no. 2 (2025): 121–136.