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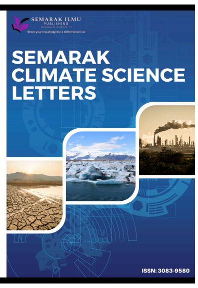

Type-2 Fuzzy Interpolation B-Spline Curve Modeling towards Earthquake Magnitude Data in Ranau, Sabah

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ABSTRACT

This research proposed a type-2 fuzzy interpolation B-spline modeling, a combination of geometric modeling, and type-2 fuzzy set theory (T2FST) to solve complex uncertainty in data. First and foremost, the complex uncertainty data is defined using the T2FST through the concept of a type-2 fuzzy number (T2FN). Along with constructing the type-2 fuzzy model, fuzzification (alpha-cut operation), reduction, and defuzzification processes are used against the type-2 fuzzy data points of earthquake magnitude in Ranau, Sabah. Then, the interpolation B-spline curve function demonstrates the type-2 fuzzy data points. Besides, the error obtained between the crisp and defuzzification values of the earthquake magnitude data in Ranau is within the acceptable range, which is less than 0.1. Therefore, this study proves that the type-2 fuzzy interpolation B-spline curve model can solve the complex uncertainty data for earthquake magnitude.

1. Introduction

The modeling process uses specific functions, either their result as in curve or data plotting. It needs a set of real data that has been collected and followed by analytical methods to obtain the best function that represents the original data. Usually, the data are collected from different resources, such as satellite images, drawings, or measurements, and then analyzed to determine the best mathematical model that represents the original data [1,2]. The selection of the appropriate mathematical model depends on the nature of the data and the application domain.

But, when the data have uncertain issues, the modeling process becomes more complex, and we are mostly unable to model them, especially in curve or surface form. Although the traditional method was applied, like getting rid of the data or applying statistical methods, it cannot solve the uncertainty problem efficiently in some cases. Therefore, another method or theory is needed to

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define the uncertainty data, whereas the defining data and other specific data can be modeled. This will give more reasonable and accurate results.

One of the methods that have been proposed to model the uncertainty data is the Fuzzy Set Theory. The fuzzy set theory was introduced by Zadeh [3] and widely applied in order to define the uncertainty before it can be extended to the specific field of research. Part of it is the geometrical modeling field. In the geometrical modeling field, fuzzy set theory has proven valuable for representing and manipulating uncertain geometric data. This is particularly relevant when dealing with imprecise measurements, vague shapes, or subjective interpretations of geometric information. For example, in defining curves and surfaces, fuzzy sets can accommodate uncertainty in control points or boundary conditions, leading to more robust and flexible geometric models [4-14].

However, when dealing with more complex uncertain data, traditional type-1 fuzzy sets (fuzzy set theory), employed in fuzzy curve and surface modeling, may be insufficient. Their inherent limitation lies in the crisp membership functions, which cannot fully capture uncertainties in the membership values themselves. In such cases, type-2 fuzzy sets (T2FS) offer a more robust approach. Unlike type-1 fuzzy sets, which are characterized by crisp membership functions, T2FS utilize fuzzy membership functions, providing a higher level of flexibility in representing uncertainties within the data [9,15]. Therefore, T2FSs provide a more suitable framework for defining and modeling complex uncertain data, addressing the limitations of crisp membership functions inherent in type-1 fuzzy sets. By employing type-2 fuzzy data (T2FD), we can generate type-2 fuzzy curves and surfaces that effectively capture the higher-order uncertainties present in the data, overcoming the limitations of traditional type-1 fuzzy set approaches. Several studies have been conducted to explore the application of type-2 fuzzy sets in geometric modeling, particularly in the context of curve and surface representations [9,15,16].

Data modeling of environmental data, sensor data, and other real-world data often face uncertainty, vagueness, and imprecision issues. This is also applied to the earthquake magnitude data. Earthquake magnitude is not a directly observable quantity. It has been calculated from measurements of seismic waves, which are inherently noisy and subject to various sources of error. The location and depth of the earthquake, the types of seismic waves recorded, and the instrumentation used all contribute to uncertainty in the magnitude estimate. Lucas et al. [17] and Musson [18] discuss magnitude uncertainties and their effects. Another key source of complex uncertainty in earthquake magnitude data is the limitation of instrumentation and inaccuracies in the collection and processing of the data. Imprecise measurements from seismic instruments, errors in locating the earthquake epicenter and depth, and issues with data processing algorithms can all contribute to uncertainty in the final magnitude estimate. These factors introduce complexities and ambiguities that must be accounted for when modeling and analyzing earthquake magnitude data [17,18].

Therefore, the complex uncertainty data of earthquake magnitude can be defined by using T2FS theory. Since the data are in numeric form, we can apply the concept of type-2 fuzzy numbers (T2FNs) to represent and model this data. T2FNs provide a more robust and flexible framework for capturing the higher-order uncertainties inherent in the earthquake magnitude data, which traditional type-1 fuzzy sets cannot fully capture. By employing T2FD, we can generate type-2 fuzzy representations, such as type-2 fuzzy curves, that effectively account for the complexities and ambiguities in the earthquake magnitude measurements. Here, the curve function that will be used is interpolation B-spline curve as the representative of T2FD which later known as type-2 fuzzy interpolation B-spline curve (T2FIBsC).

Although the T2FIBsC of earthquake magnitude data can be developed, but the data modeling is not as a final answer because we need a set of single data point to represent the final result together with its curve. To achieve this, we need to implement a defuzzification process to convert the type-2 fuzzy interpolation B-spline curve into a single crisp curve that represents the earthquake magnitude data with its uncertainty information.

After the complex uncertainty data had been defined and characterized as type-2 fuzzy data points (T2FDPs), the fuzzification, alpha-cut, type-reduction, and defuzzification processes will then be implemented to obtain the final crisp representation of the T2FDPs. The fuzzification process is to transform the numeric earthquake magnitude data in Ranau, Sabah into a T2FN representation, where each data point is expressed as a T2FDPs. The alpha-cut process is then applied to extract the upper and lower bounds of the T2FDPs, which represent the range of possible values for each data point with the specific alpha value determined by the user. The type-reduction process involves applying an appropriate type-reduction method, such as the mean method of interval T2FDPs after alpha-cut process, to convert the T2FDPs into a type-1 fuzzy set, which allow the defuzzification process of type-1 fuzzy set can be implemented directly because there is no defuzzification of T2FS can be used. Finally, a type-1 fuzzy defuzzification method, such as the mean method, is applied to the type-reduced fuzzy set to obtain a single crisp value for each data point.

For each processes, the new T2FDPs will be modeled using an interpolation B-spline curve function [19-21]. The interpolation B-spline curve function will generate the earthquake magnitude data in Ranau, Sabah, obtained from the Pusat Kajian Bencana Alam, Universiti Malaysia Sabah. The collected data was subject to various sources of error. When collecting and recording the seismographic measurements of earthquake magnitude, a seismometer is used to measure the position with a high probability of active seismic activity. The site of the seismograph and seismometer is often situated in the highlands or areas where earthquake shaking is prevalent. However, one factor contributing to errors in the earthquake magnitude data is the decision-making process regarding the placement of the seismic instrumentation.

This paper is organized as follows: Section 2 discusses the representation of data points using T2FST, the T2FN concept, and interpolation of the B-spline curve function. Then, Section 3 will discuss the fuzzification, type-reduction, and defuzzification method processes of T2FDPs. Next, Section 4 will show the application of the T2FIBsC model through the proposed method discussed in Section 3 to model the earthquake magnitude in Ranau, Sabah, from a set of complex uncertainty data. In addition, this section will also compare the crisp data with the defuzzification data to show the effectiveness of the proposed T2FIBsC model.

2. Methodology

This section defines complex uncertainty data based on the T2FST through the concept of T2FN. The T2FN concept, which is implemented from T2FST, will be used to solve the problem of complex uncertainty data.

2.1 Type-2 Fuzzy Set Theory

Definition 1. A type-2 fuzzy set is denoted by $\ddot{\overleftrightarrow{A}}$, is characterized by a type-2 membership function $\mu_{\ddot{\overleftrightarrow{A}}}(x,u)$ where $x \in X$ and $\mu \in J_x \subseteq [0,1]$, that is

$$\ddot{\overleftrightarrow{A}} = \left\{ \left((x,u), \mu_{\ddot{\overleftrightarrow{A}}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \tag{1}$$

in which $0 \le \mu_{\ddot{\tilde{A}}}(x,u) \le 1$. $\ddot{\tilde{A}}$ can also be defined as

$$\ddot{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\ddot{\tilde{A}}}(x,u) \big/ (x,u) J_x \subseteq [0,1] \tag{2}$$

where $\iint$ represent the union of all x and u that can be accepted. J_x is the primer member of x. In other words, x is the primer domain while J_x is the secondary domain [22].

2.2 Type-2 Fuzzy Number

Definition 2. A T2FN is broadly defined as a T2FS that has a numerical domain. An interval T2FS is defined using the following four constraints, where $\ddot{\tilde{A}}_\alpha = \left\{[a^\alpha, b^\alpha],[c^\alpha, d^\alpha]\right\}$, $\forall \alpha \in [0,1]$, $\forall a^\alpha, b^\alpha, c^\alpha, d^\alpha \in \sim$ [19].

1. $a^\alpha \le b^\alpha \le c^\alpha \le d^\alpha$.
2. $[a^\alpha, d^\alpha]$ and $[b^\alpha, c^\alpha]$ generate a function that is convex and $[a^\alpha, d^\alpha]$ generate a function is normal.
3. $\forall \alpha_1, \alpha_2 \in [0,1] : (\alpha_2 > \alpha_1) \Rightarrow \left(\left[a^{\alpha_1}, c^{\alpha_1}\right] \supset \left[a^{\alpha_2}, c^{\alpha_2}\right], \left[b^{\alpha_1}, d^{\alpha_1}\right] \supset \left[b^{\alpha_2}, d^{\alpha_2}\right]\right)$, for $c^{\alpha_2} \ge b^{\alpha_2}$.
4. If the maximum of the membership function generated by $[b^\alpha, c^\alpha]$ is the level α_m, that is, $[b^{\alpha_m}, c^{\alpha_m}]$, then $\left[b^{\alpha_m}, c^{\alpha_m}\right] \subset \left[a^{\alpha=1}, d^{\alpha=1}\right]$.

Figure 1 shows the interval of T2FN in triangular form. The footprint of uncertainty (FOU) is bounded by two triangular type-1 fuzzy sets known as upper membership function and lower membership function.

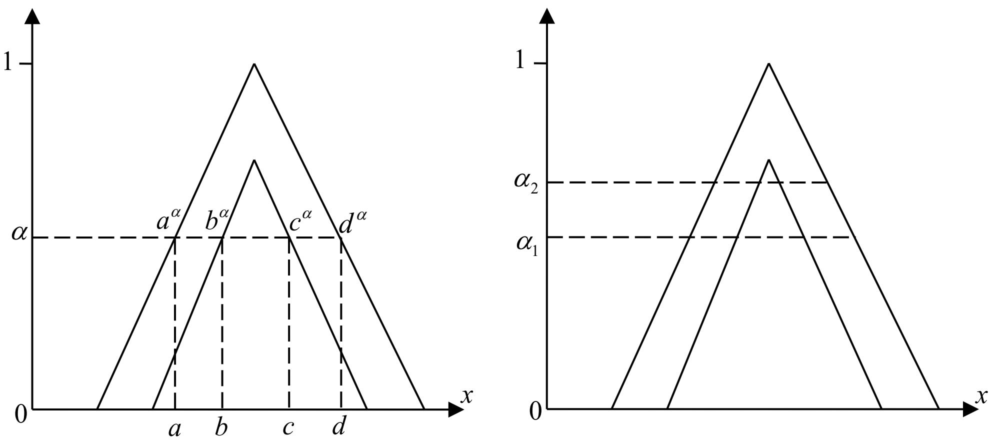

Fig. 1. Definition of an interval T2FN

2.3 Type-2 Fuzzy Data Points

Definition 3. Let $P=\{x \mid x \text{ type-2 fuzzy point}\}$ and $\overleftrightarrow{P}=\{P_i \mid P_i \text{ data point}\}$ which is set of type-2 fuzzy data point with $P_i \in P \subset X$, where *X* is a universal set and $\mu_P(P_i): P \rightarrow [0,1]$ is the membership function which defined as $\mu_P(P_i)=1$ and formulated by $\overleftrightarrow{P}=\{(P_i, \mu_P(P_i)) \mid P_i \in {}^{\sim}\}$. Therefore,

$$\mu_P(P_i)=\begin{cases} 0 & \text{if } P_i \notin X \\ c \in (0,1) & \text{if } P_i \,\tilde{\in}\, X \\ 1 & \text{if } P_i \in X \end{cases} \tag{3}$$

with $\mu_P(P_i)=\left\langle \mu_P(\overleftrightarrow{P}_i^{\leftarrow}), \mu_P(P_i), \mu_P(\overleftrightarrow{P}_i^{\rightarrow}) \right\rangle$ which $\mu_P(\overleftrightarrow{P}_i^{\leftarrow})$ and $\mu_P(\overleftrightarrow{P}_i^{\rightarrow})$ are left and right footprint of membership values with $\mu_P(\overleftrightarrow{P}_i^{\leftarrow})=\left\langle \mu_P(\vec{P}_i^{\overleftarrow{\leftarrow}}), \mu_P(\vec{P}_i^{\leftarrow}), \mu_P(\vec{P}_i^{\overrightarrow{\leftarrow}}) \right\rangle$ where, $\mu_P(\vec{P}_i^{\overleftarrow{\leftarrow}})$, $\mu_P(\vec{P}_i^{\leftarrow})$ and $\mu_P(\vec{P}_i^{\overrightarrow{\leftarrow}})$ are left-left, left, right-left membership grade values and $\mu_P(\vec{P}_i^{\overrightarrow{\rightarrow}})$, $\mu_P(\vec{P}_i^{\rightarrow})$ and $\mu_P(\vec{P}_i^{\overleftarrow{\rightarrow}})$ are right-right, right, left-right membership grade values, which can be written as

$$\overleftrightarrow{P}=\left\{\overleftrightarrow{P}_i : i=0,1,2,\ldots,n\right\} \tag{4}$$

for every *i*, $\overleftrightarrow{P}_i=\left\langle \overleftrightarrow{P}_i^{\leftarrow}, P_i, \overleftrightarrow{P}_i^{\rightarrow} \right\rangle$ with $\overleftrightarrow{P}_i^{\leftarrow}=\left\langle \vec{P}_i^{\overleftarrow{\leftarrow}}, \vec{P}_i^{\leftarrow}, \vec{P}_i^{\overrightarrow{\leftarrow}} \right\rangle$ where $\vec{P}_i^{\overleftarrow{\leftarrow}}$, $\vec{P}_i^{\leftarrow}$ and $\vec{P}_i^{\overrightarrow{\leftarrow}}$ are left-left, left and right-left T2FDPs and $\overleftrightarrow{P}_i^{\rightarrow}=\left\langle \vec{P}_i^{\overleftarrow{\rightarrow}}, \vec{P}_i^{\rightarrow}, \vec{P}_i^{\overrightarrow{\rightarrow}} \right\rangle$ where $\vec{P}_i^{\overleftarrow{\rightarrow}}$, $\vec{P}_i^{\rightarrow}$ and $\vec{P}_i^{\overrightarrow{\rightarrow}}$ are left-right, right and right-right T2FDPs respectively. This can be illustrated as in Figure 2. The illustration of T2FDP was shown in Figure 2 which type-1 fuzzy data point (T1FDP) becomes the primary membership function bounded by upper bound, $\left[\vec{P}^{\overleftarrow{\leftarrow}}, P, \vec{P}^{\overrightarrow{\rightarrow}}\right]$ and lower bound, $\left[\vec{P}^{\overrightarrow{\leftarrow}}, P, \vec{P}^{\overleftarrow{\rightarrow}}\right]$ respectively. The process of defining T2FDP can be shown through Figure 3.

Figure 2 shows that the F2FDP around 5. The type-1 triangular fuzzy data bounded by upper and lower fuzzy data. This T2FDP also known as normal T2FDP since the full membership for upper and lower membership are not equal.

Figure 3 illustrates the process of defining type-2 fuzzy data points (T2FDPs) from ordinary data points. The T2FDPs are formed based on the definition and properties of type-2 fuzzy numbers (T2FNs).

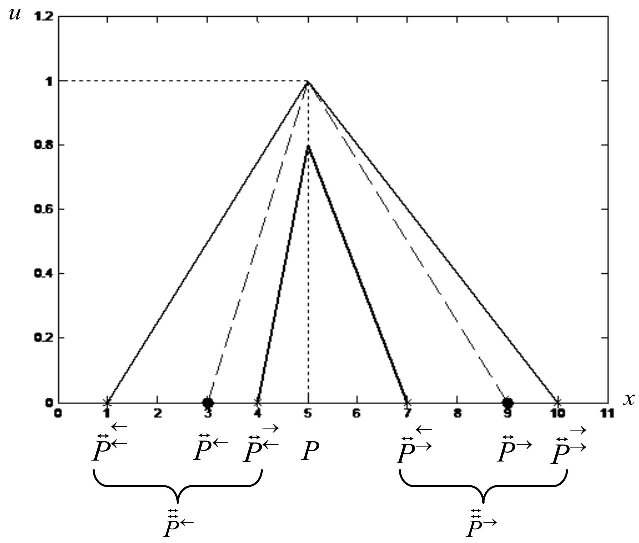

Fig. 2. T2FDP around 5

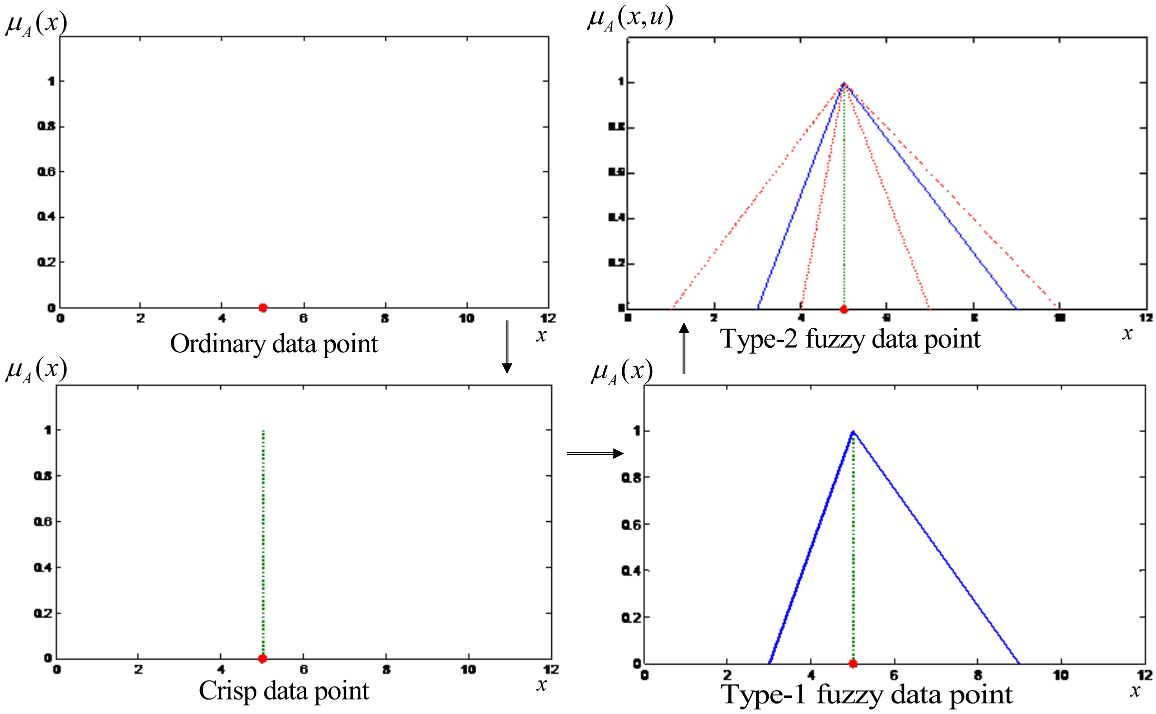

Fig. 3. Process of defining T2FDP

2.4 Alpha-Cut Method

Definition 4. Let $\ddot{\vec{P}}$ be the set of T2FDPs with $\ddot{\vec{P}}_i \in \ddot{\vec{P}}$ where $i = 0,1,...,n-1$. Then $\ddot{\vec{P}}^{\alpha}$ is the alpha-cut operation of T2FDPs which is given as equation as follows.

$$\begin{aligned}\overleftrightarrow{\overleftrightarrow{P}}_i^{\alpha} &= \left\langle \overleftrightarrow{\overleftrightarrow{P}}_i^{\alpha\leftarrow}, P_i, \overleftrightarrow{\overleftrightarrow{P}}_i^{\alpha\rightarrow} \right\rangle \\ &= \left\langle \left\langle \overleftarrow{\overleftrightarrow{P}_i^{\alpha\leftarrow}}; \overleftrightarrow{P}_i^{\alpha\leftarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\alpha\leftarrow}} \right\rangle, P_i, \left\langle \overleftarrow{\overleftrightarrow{P}_i^{\alpha\rightarrow}}; \overleftrightarrow{P}_i^{\alpha\rightarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\alpha\rightarrow}} \right\rangle \right\rangle \\ &= \left\langle \left[\left(P_i - \left\langle \overleftarrow{\overleftrightarrow{P}_i^{\leftarrow}}; \overleftrightarrow{P}_i^{\leftarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\leftarrow}} \right\rangle \right)\alpha + \left\langle \overleftarrow{\overleftrightarrow{P}_i^{\leftarrow}}; \overleftrightarrow{P}_i^{\leftarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\leftarrow}} \right\rangle \right], P_i, \right. \\ &\quad \left. \left[-\left(\left\langle \overleftarrow{\overleftrightarrow{P}_i^{\rightarrow}}; \overleftrightarrow{P}_i^{\rightarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\rightarrow}} \right\rangle - P_i \right)\alpha + \left\langle \overleftarrow{\overleftrightarrow{P}_i^{\rightarrow}}; \overleftrightarrow{P}_i^{\rightarrow}; \overrightarrow{\overleftrightarrow{P}_i^{\rightarrow}} \right\rangle \right] \right\rangle \end{aligned} \tag{5}$$

This definition can be illustrated through Figure 4. Figure 4 shows that the alpha-cut of a T2FDP at level alpha represents the range of T2FDPs that have a secondary membership grade of at least alpha. A higher alpha value corresponds to a stricter membership grade requirement, resulting in a narrower alpha-cut interval. This interval can be interpreted as a range of plausible data values, with a higher confidence level associated with higher alpha values.

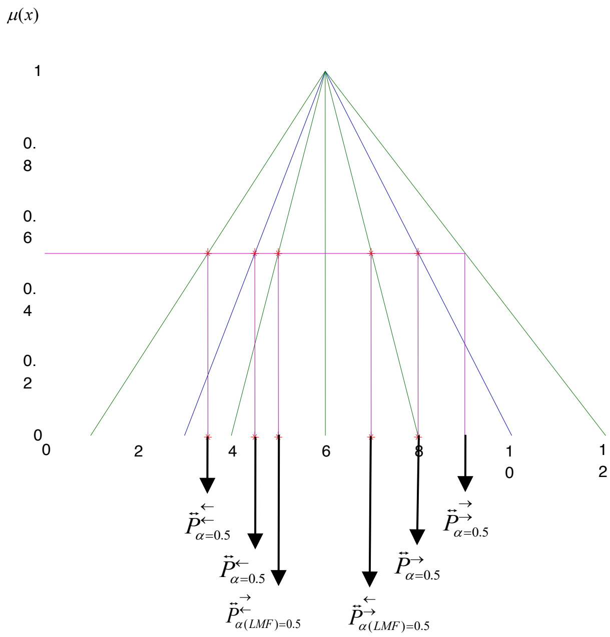

Fig. 4. The alpha-cut operation towards T2FDPs

The subsequent step involves the type-reduction method after applying the fuzzification process on the T2FDPs. The type-reduction technique transforms the T2FDPs into type-1 fuzzy data points (T1FDPs) after the fuzzification process. Additionally, type-reduction is utilized to enable the application of type-1 defuzzification methods. Numerous type-reduction approaches have been discussed in the literature [20,21]. However, this paper introduces a novel type-reduction method known as the centroid min method, as described in Definition 5.

2.5 Type-Reduction Method

Definition 5. Let $\overleftrightarrow{\overleftrightarrow{P}}_i$ be a set $(n+1)$ T2FDPs, then type-reduction method of alpha-T2FDPs (after fuzzification), $\overline{\overleftrightarrow{P}}_i$ is defined by

$$\overline{\overline{P}}^{\alpha}=\left\{\overline{\overline{P}}_i^{\alpha}=\left\langle \overline{\overleftrightarrow{P}}_i^{\alpha\leftarrow},P_i,\overline{\overleftrightarrow{P}}_i^{\alpha\rightarrow}\right\rangle ; i=0,1,\ldots,n\right\} \tag{6}$$

where $\overline{\overleftrightarrow{P}}_i^{\alpha\leftarrow}$ is left type-reduction of alpha-cut T2FDPs, $\overline{\overleftrightarrow{P}}_i^{\alpha\leftarrow}=\frac{1}{3}\sum_{i=0,\ldots,n}\left\langle \overleftarrow{\vec{P}}_i^{\alpha\leftarrow}+\vec{P}_i^{\alpha\leftarrow}+\overrightarrow{\vec{P}}_i^{\alpha\leftarrow}\right\rangle$, P_i is the crisp point and $\overline{\overleftrightarrow{P}}_i^{\alpha\rightarrow}$ is right type-reduction of alpha-cut T2FDPs, $\overline{\overleftrightarrow{P}}_i^{\alpha\rightarrow}=\frac{1}{3}\sum_{i=0,\ldots,n}\left\langle \overleftarrow{\vec{P}}_i^{\alpha\rightarrow}+\vec{P}_i^{\alpha\rightarrow}+\overrightarrow{\vec{P}}_i^{\alpha\rightarrow}\right\rangle$.

2.6 Defuzzification Method

Definition 6. Let alpha-TR is the type-reduction method after alpha-cut process had been applied for every T2FDPs, $\overleftrightarrow{P}_i^{\alpha}$. Then $\overline{\overline{P}}_i^{\alpha}$ named as defuzzification T2FDPs for $\overleftrightarrow{P}_i^{\alpha}$ if for every $\overleftrightarrow{P}_i^{\alpha}\in\overleftrightarrow{P}^{\alpha}$,

$$\overline{\overline{P}}^{\alpha}=\left\{\overline{\overline{P}}_i^{\alpha}\right\} \text{ for } i=0,1,\ldots,n \tag{7}$$

where for every $\overline{\overline{P}}_i^{\alpha}=\frac{1}{3}\sum_{i=0}<\overline{\overleftrightarrow{P}}_i^{\alpha\leftarrow},P_i,\overline{\overleftrightarrow{P}}_i^{\alpha\rightarrow}>$. The process in defuzzifying T2FDPs can be illustrated at Figure 5.

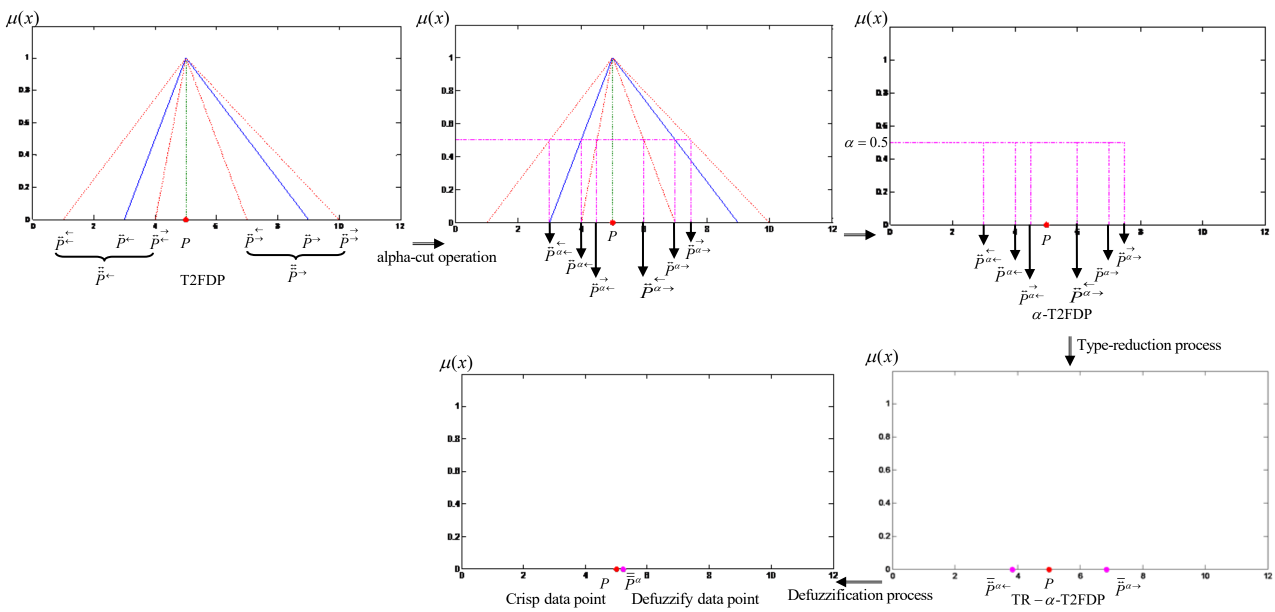

Fig. 5. Defuzzification process of T2FDP

Figure 5 shows that the defuzzification process of T2FDP. This process led by fuzzification process first where the complex uncertainty data had been defined as T2FDP. After that, the alpha-cut method is applied where the interval of T2FDP become smaller depends on the increasing value of alpha. Then, type-reduction is applied to convert T2FDP after alpha-cut become type-1 fuzzy data which will allow the defuzzification method of type-1 fuzzy can be implemented directly.

When we have finished defining the complex uncertainty data points using type-2 fuzzy set theory, known as T2FDPs, we want to illustrate them through a curve by using an interpolation B-spline curve function. This can be known as a type-2 fuzzy interpolation B-spline curve (T2FIBsC), which will give us a more comprehensive understanding of how T2FDPs can be modeled, providing us with the curves' shapes based on the data points. Besides that, we can understand the meaning of the T2FDPs based on the yielded curves.

2.7 Type-2 Fuzzy Interpolation B-Spline Curve

Definition 7. Let $\overset{\leftrightarrow}{\ddot{d}}_i \in R^m$ be a list of T2FDPs with $0 \le i \le n$, then the T2FIBsC can be defined as

$$\overleftrightarrow{BsC}(t) = \sum_{i=0}^{k+h-1} \overset{\leftrightarrow}{\ddot{P}}_i B_{i,k}(t), \quad \text{which} \quad \overleftrightarrow{BsC}(t_i) = \overset{\leftrightarrow}{\ddot{d}}_i, \tag{8}$$

where t is crisp knot sequences $t_1, t_2, \ldots, t_{m=k+2(n-1)}$, $\overset{\leftrightarrow}{\ddot{P}}_i$ are type-2 fuzzy control points(T2FCPs) where early before $\overset{\leftrightarrow}{\ddot{P}}_i$ are T2FDPs but here are T2FCPs with the same properties as mentioned in definitions before. The $B_{i,k}(t)$ is basic function of B-spline. In this part, T2FCPs need to find first based on the T2FDPs which it will force the constructed curve to interpolate the T2FDPs. Here, the T2FDPs known as $\overset{\leftrightarrow}{\ddot{d}}_i$.

Therefore, to illustrate the T2FIBsC, we summarized in the table form (Table 1) for the computed T2FDPs and also in curve form by looking at Figure 6 as follows.

Table 1 shows the numerical example of the fuzzification, type-reduction, and defuzzification processes for the T2FDPs. Based on this table, we will illustrate these processes in curve form using the interpolation B-spline curve function. Specifically, we will model the complex uncertainty data points using the T2FDP representation and then generate the corresponding B-spline curves as T2FIBsC to provide a more comprehensive understanding of how T2FDPs can be represented and analyzed graphically.

Table 1
Process of fuzzification, type-reduction and defuzzification of T2FIBsC

T2FDPs $\overleftrightarrow{BsC}(t)=\overset{\leftrightarrow}{d}_i$	$\overset{\leftrightarrow}{d}_i$						
	$\overset{\leftrightarrow}{d}_i^{\overset{\leftarrow}{\leftarrow}}$	$\overset{\leftrightarrow}{d}_i^{\leftarrow}$	$\overset{\leftrightarrow}{d}_i^{\overset{\rightarrow}{\leftarrow}}$	d_i	$\overset{\leftrightarrow}{d}_i^{\overset{\leftarrow}{\rightarrow}}$	$\overset{\leftrightarrow}{d}_i^{\rightarrow}$	$\overset{\leftrightarrow}{d}_i^{\overset{\rightarrow}{\rightarrow}}$
$i=0$	(-12, 0)	(-11, 0)	(-9, 0)	(-5, 0)	(3, 0)	(6,0)	(9,0)
$i=1$	(15, 28)	(15, 26)	(15, 25)	(15, 20)	(15, 16)	(15,14)	(15,12)
$i=2$	(17, -13)	(15, -15)	(13, -17)	(10, -20)	(8, -22)	(5,-25)	(3,-27)
$i=3$	(30, 10)	(32, 10)	(34, 10)	(40, 10)	(46, 10)	(48,10)	(49,10)
Alpha-Cut, $\alpha=0.5$ $\overleftrightarrow{BsC}^{\alpha}(t)=\overset{\leftrightarrow}{d}_i^{\alpha}$	$\overset{\leftrightarrow}{d}_i^{\alpha}$						
	$\overset{\leftrightarrow}{d}_i^{\overset{\leftarrow}{\alpha\leftarrow}}$	$\overset{\leftrightarrow}{d}_i^{\alpha\leftarrow}$	$\overset{\leftrightarrow}{d}_i^{\overset{\rightarrow}{\alpha\leftarrow}}$	d_i	$\overset{\leftrightarrow}{d}_i^{\overset{\leftarrow}{\alpha\rightarrow}}$	$\overset{\leftrightarrow}{d}_i^{\alpha\rightarrow}$	$\overset{\leftrightarrow}{d}_i^{\overset{\rightarrow}{\alpha\rightarrow}}$
$i=0$	(-8.5, 0)	(-8, 0)	(-7, 0)	(-5, 0)	(-1, 0)	(0.5, 0)	(2, 0)
$i=1$	(15, 24)	(15, 23)	(15, 22.5)	(15, 20)	(15, 18)	(15, 17)	(15, 16)
$i=2$	(13.5, -16.5)	(12.5, -17.5)	(11.5, -18.5)	(10, -20)	(9, -21)	(7.5, -22.5)	(6.5, -23.5)
$i=3$	(35, 10)	(36, 10)	(37, 10)	(40, 10)	(43, 10)	(44, 10)	(44.5, 10)
Type-Reduction $\overline{\overleftrightarrow{BsC}}^{\alpha}(t)=\overset{\leftrightarrow}{\bar{d}}_i^{\alpha}$	$\overset{\leftrightarrow}{\bar{d}}_i^{\alpha}$						
	$\overset{\leftrightarrow}{\bar{d}}_i^{\alpha\leftarrow}$			d_i	$\overset{\leftrightarrow}{\bar{d}}_i^{\alpha\rightarrow}$		
$i=0$	(-7.8333, 0)			(-5, 0)	(0.5, 0)		
$i=1$	(15, 23.1667)			(15, 20)	(15, 7)		
$i=2$	(12.5, -17.5)			(10, -20)	(7.6667, -22.3333)		
$i=3$	(36, 10)			(40, 10)	(48.8333, 10)		
Type-2 Defuzzification $\overline{\overline{BsC}}^{\alpha}(t)=\bar{\bar{d}}_i^{\alpha}$				$\bar{\bar{d}}_i^{\alpha}$			
				d_i	$\bar{\bar{d}}_i^{\alpha}$		
$i=0$				(-5, 0)	(-4.1111, 0)		
$i=1$				(15, 20)	(15, 20.0556)		
$i=2$				(10, -20)	(10.0556, -19.9444)		
$i=3$				(40, 10)	(39.9444, 10)		

Figure 6 illustrates the comprehensive process of modeling Perfectly Normal Type-2 Fuzzy Data Points (PNT2FDPs) using the interpolation B-spline curve function. This process involves several key steps:

i. Modeling the T2FDPs to capture the complex uncertainty inherent in the data.
ii. Generating the Type-2 Fuzzy Interpolation B-spline Curve (T2FIBsC) based on the modeled T2FDPs.
iii. Applying defuzzification techniques to the T2FDPs to obtain the final crisp representation of the T2FDPs solution.

This step-by-step approach provides a detailed understanding of how T2FDPs can be effectively modeled and represented using the powerful capabilities of the interpolation B-spline curve function.

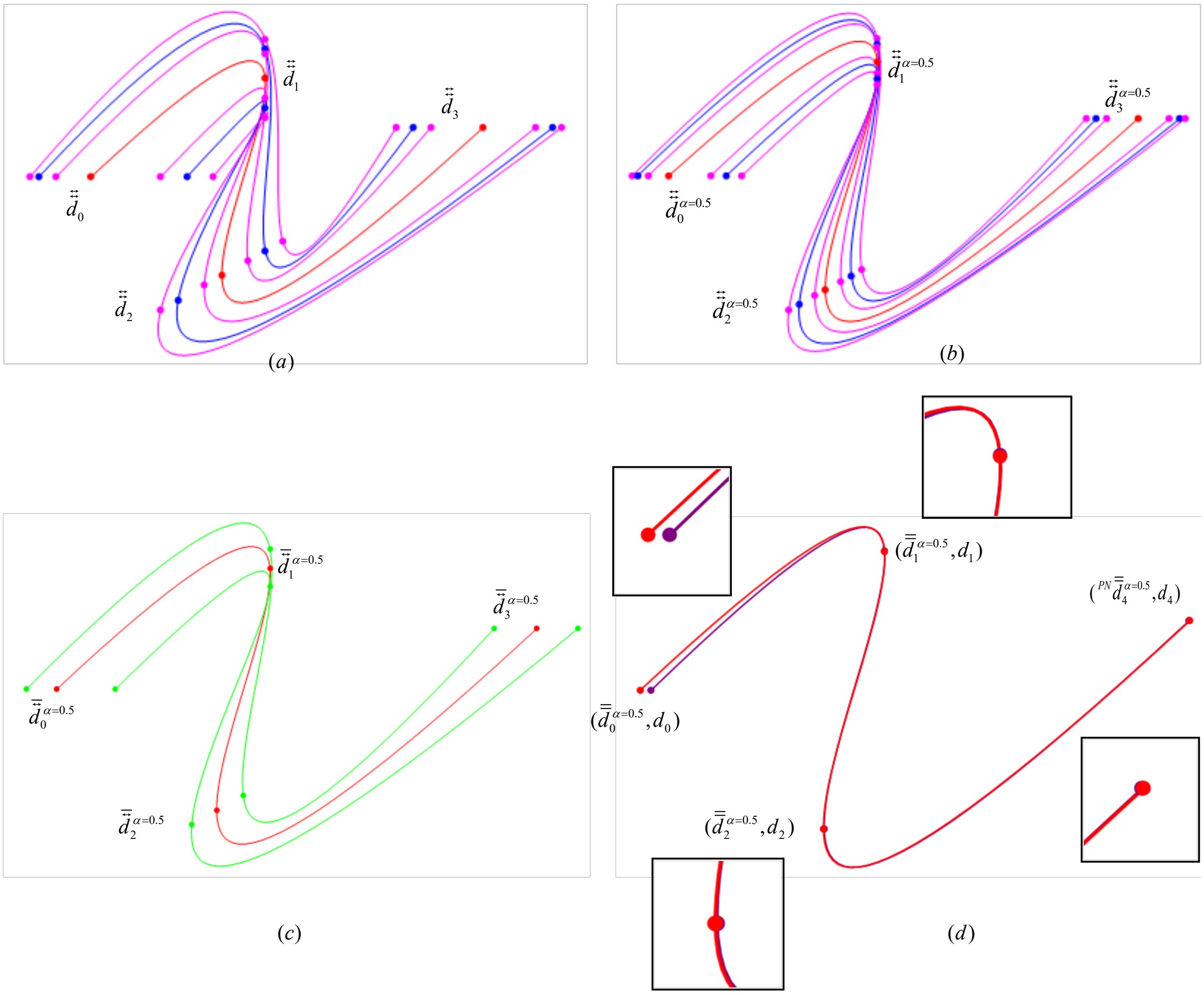

Fig. 6. The modeling of (a) T2FIBsC, (b) alpha-cut operation of T2FIBsC(α -T2FIBsC), (c) type-reduction of α -T2FIBsC(TR- α -T2FIBsC) and (d) defuzzification of TR- α -T2FIBsC

3. Results

This section illustrates an application example of the T2FIBsC model for representing and analyzing the earthquake magnitude data collected in Ranau, Sabah. The data set consists of eight earthquake magnitude measurements plotted to model the T2FIBsC. By leveraging the concepts of T2FST and T2FNs discussed in the previous section, the T2FIBsC model is constructed to provide a comprehensive visual representation of the earthquake magnitude data, as shown in Figure 7.

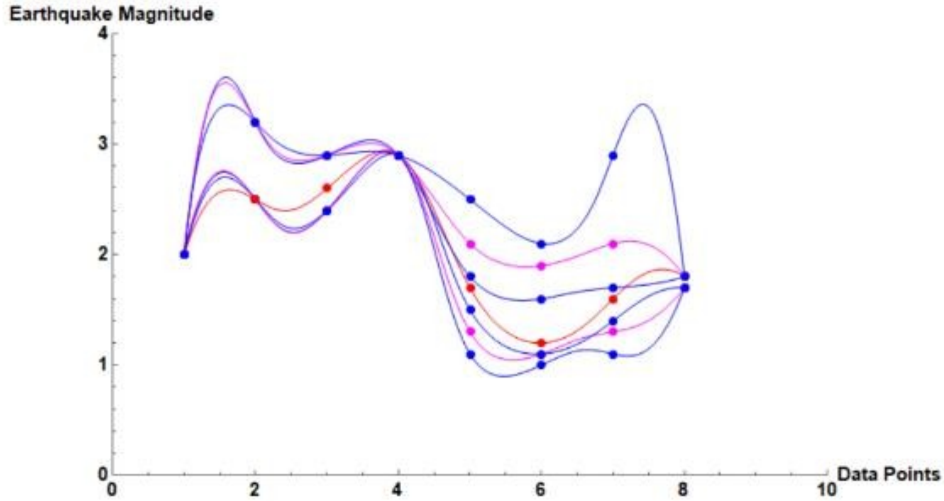

Fig. 7. T2FIBsC model of earthquake magnitude data collected in Ranau, Sabah

After achieving the T2FIBsC model, the next step is to apply the alpha-method process, as shown in Figure 8. This process utilizes Definition 4 and Eq. (5), with the α-value set to 0.5. Applying this step will reduce the interval between the T2FDPs and the crisp data points. This helps to bridge the gap between the fuzzy and non-fuzzy representations of the data, providing a more cohesive and comprehensive understanding of the underlying uncertainty.

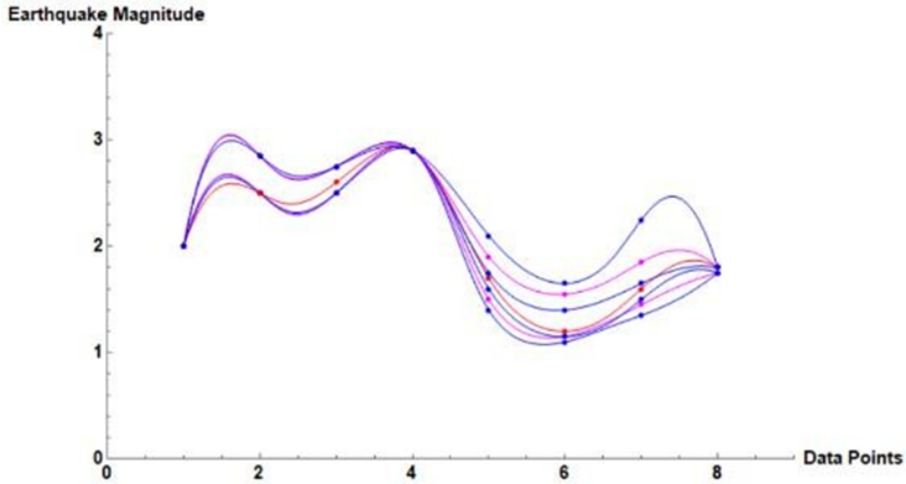

Fig. 8. Fuzzification model of T2FIBsC of earthquake magnitude data

After the alpha-cut process defined in Definition 4, the T2FDPs will be transformed and reduced to type-1 fuzzy data points (T1FDPs) through the type-reduction process described in Definition 5. This type-reduction step is a crucial part of the overall process, as it allows the complex type-2 fuzzy data points to be represented in a more straightforward type-1 fuzzy form, making them more accessible and easier to work with. Figure 9 then shows the type-reduction model. It illustrates how the type-2 fuzzy interpolation B-spline curve model is transformed into its corresponding type-1 representation, providing a more streamlined and interpretable form of the fuzzy data points.

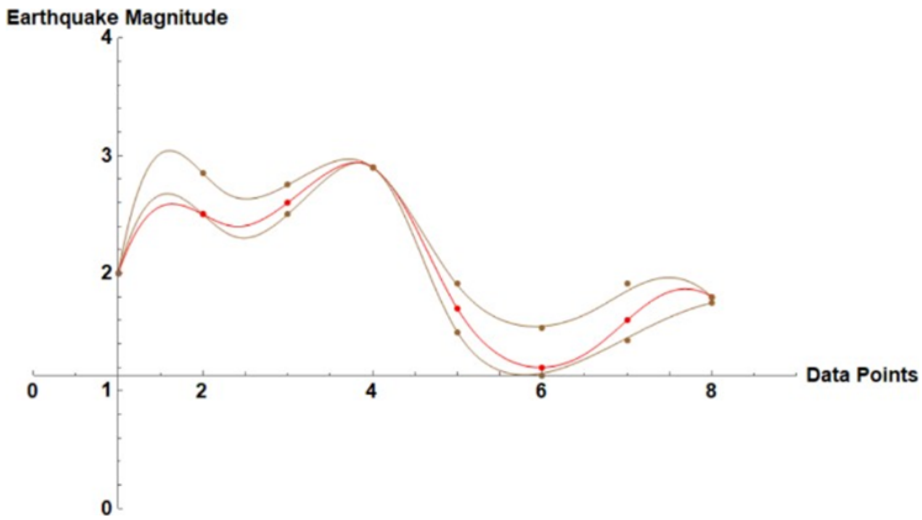

Fig. 9. Type-reduction model of T2FIBsC of earthquake magnitude data

Finally, Figure 10 illustrates the defuzzification process of the earthquake magnitude data. This process obtains a crisp data value based on the defuzzification step described in Definition 6. The defuzzification transforms the type-1 fuzzy data points into a single representative value, providing a concise and interpretable representation of the complex fuzzy data.

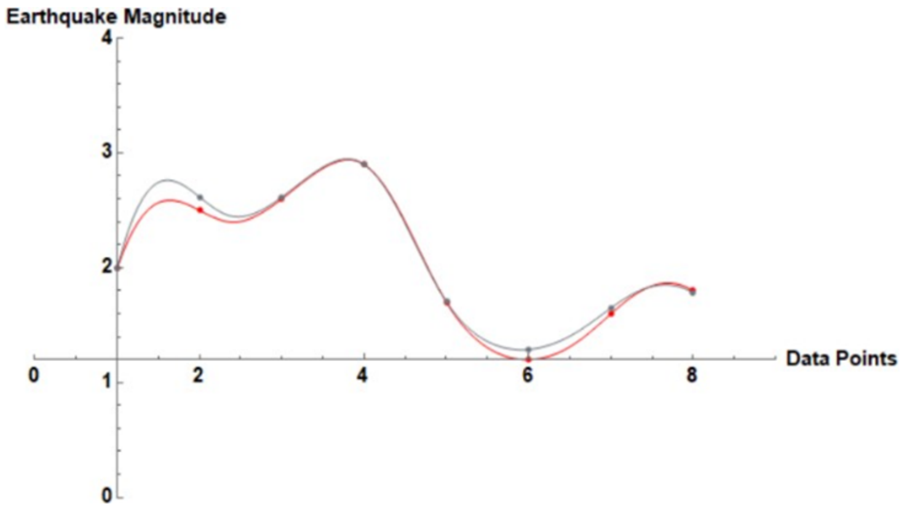

Fig. 10. Defuzzification model of T2FIBsC of earthquake magnitude data

Figure 10 shows that the result after applying the defuzzification process to the T2FIBsC is a set of crisp data points that can be used for further analysis and applications. The red curve represents a crisp curve of earthquake magnitude data modeled through the T2FIBsC. The grey curve also represents a defuzzification curve of earthquake magnitude data, showing slightly different values between those two curves and the data points.

In addition, in Figures 7 and 8, the red curve represents the crisp data points. The blue curve represents the left-left T2FDPs, left-right T2FDPs, right-right T2FDPs, and right-left T2FDPs. Meanwhile, the magenta curve represents the right T2FDPs and the left T2FDPs. In Figure 9, the brown curve represents the left and right T2FDPs, and the red curve represents the crisp data points. The figures visually represent the various types of fuzzy data points and their corresponding curves, allowing for a more comprehensive understanding of the modeling process.

The errors between the crisp data points and the defuzzification data points, calculated using Eq. (9), are analyzed to investigate the effectiveness and accuracy of the earthquake magnitude data output. Figure 11 presents a detailed comparison between the crisp data points and the defuzzification data points of the earthquake magnitude, providing valuable insights into the performance and reliability of the modeling approach.

$$\frac{\sum_{i=0}^{n} \overline{\overline{d}}_{i_e}}{n} \text{ with } \overline{\overline{d}}_{i_e} = \frac{\left|\overline{\overline{d}}_i - d_i\right|}{\overline{\overline{d}}_i},\ i = 0,1,2,\ldots,n \text{ and } n = 7 \tag{9}$$

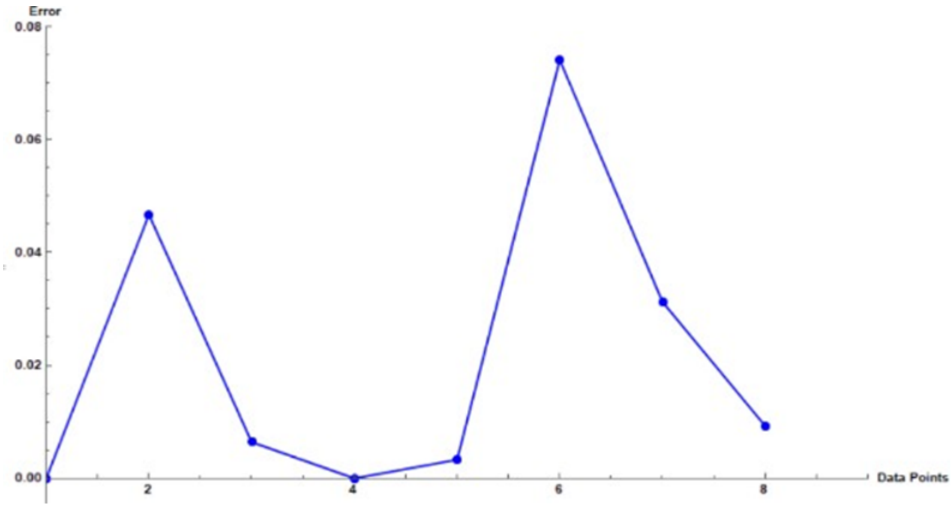

Fig. 11. The comparison between the crisp data points and defuzzify data points of earthquake magnitude data in Ranau, Sabah

The average error for both models is 0.02706244, which is significantly less than the acceptable error threshold of 0.1. This indicates that the T2FIBsC model provides a highly accurate representation of the earthquake magnitude data, and can be confidently accepted as a valid model. The low error value suggests that the type-2 fuzzy approach effectively captures the complex uncertainty inherent in the seismic measurements, leading to improved accuracy in the final earthquake magnitude estimates.

The T2FIBsC of earthquake magnitude data against all process to obtain the final result had been explain and illustrate through figures before. The following is the algorithm of the T2FIBsC of earthquake magnitude data which give more understanding on how this model work and applied towards the earthquake magnitude data.

Algorithm 1. T2FIBsC modeling of earthquake magnitude data.

Step 1: *Collect data from the data center of Ranau earthquake magnitude data.*

Step 2: *Define the uncertainty complex uncertainty data of earthquake magnitude using Definition 3.*

Step 3: *Find the type-2 fuzzy control points based on Eq. (8) and given T2FDPs.*

Step 4: *Model the T2FDPs based on Eq. (8) and the result at Figure 7.*

Step 5: *Alpha-cut process: Apply Eq. (5) against the T2FDP of earthquake magnitude data, and the result is shown in Figure 8.*

Step 6: *Type-reduction process: Apply Definition 5 against the T2FDP of earthquake magnitude data, and the result is shown in Figure 9.*

Step 7: *Defuzzification process: Apply Definition 6 against the T2FDP of earthquake magnitude data, and the result is shown in Figure 10.*

Step 8: *Find and model the error between defuzzification T2FDPs and crisp data points of earthquake magnitude data by applying Eq. (9). The result is shown in Figure 11.*

4. Conclusions

In conclusion, a T2FIBsC model has been constructed based on the concept of T2FST and the B-spline curve function. The main contribution of this method is to define the complex uncertainty data of earthquake magnitude in Ranau, Sabah while modeling the data using the B-spline curve function.

When the T2FDPs was successfully defined, the fuzzification process, reduction, and defuzzification processes were applied. These methods have their definitions to hold the crisp type-2 fuzzy solution as a final result. Thus, in order to illustrate the T2FDPs for better understanding, the T2FDPs is blended with the interpolation B-spline curve function to produce the type-2 fuzzy interpolation B-spline curve model.

The fuzzification, type-reduction, and defuzzification processes are critical components of this framework, as they facilitate the transformation of complex T2FDPs into a more accessible and manageable type-1 fuzzy form. This transformation is a vital step in the overall procedure, as it enables the representation of the intricate T2FDPs in a more straightforward and interpretable type-1 fuzzy form. As illustrated in Figure 9, the type-reduction model demonstrates how the type-2 fuzzy interpolation B-spline curve model is transformed into its corresponding type-1 representation, providing a more streamlined and interpretable form of the fuzzy data points that can be more readily comprehended and utilized for further analysis and applications.

Furthermore, the defuzzification process, as shown in Figure 10, is applied to the earthquake magnitude data to obtain a single data value based on Definition 6. This final step ensures that the complex fuzzy data can be effectively utilized for further analysis and applications.

In addition, these T2FDPs can be applied in defining complex uncertainty data, which can be modeled through various curve functions or surface functions in approximation forms such as Bezier, rational Bezier, rational B-spline, and NURBS functions. This allows for a more comprehensive and flexible representation of the complex data, enabling deeper insights and more accurate modeling of the earthquake magnitude data in Ranau, Sabah.

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