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# Analytical Study of Non-Newtonian Magnetic Casson Blood Flow with Gold Nanoparticles Through an Inclined Stenosed Artery

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### ABSTRACT

This study presents a mathematical model of non-Newtonian Casson blood flow in an inclined stenosed artery with the inclusion of gold nanoparticles. The governing fractional differential equations are formulated using the Caputo-Fabrizio fractional derivative without a singular kernel. The blood flow was modelled under the influence of a uniformly distributed magnetic field and an oscillating pressure gradient for both blood and magnetic particles velocity profiles. The analytical solutions were obtained using Laplace and finite Hankel transforms. These distributions are presented graphically using Mathcad software to analyse the effects of various physical parameters. To validate the results, the obtained solutions in limiting cases were compared with previously published findings and showed good agreement. The analysis reveals that the velocities of both blood and magnetic particles of both blood and magnetic particles increase with increasing fractional parameter, Casson parameter and time, whereas both velocity decrease as the Hartmann number increase. This model provides valuable insights into the hemodynamic behaviour of blood flow, particularly in understanding and treating cardiovascular diseases such as atherosclerosis.

## 1. Introduction

Blood is an essential fluid that circulates through the blood vessels, supporting the major physiological functions of the human body. As a complex biofluid, it transports oxygen and nutrients to tissues while simultaneously removing metabolic waste products. Sharma *et al.*, [1] reported that red blood cells (RBCs) are rich in haemoglobin, an iron-containing protein that contributes to the magnetic properties of blood depending on its oxygenation state. Javeed and Jung [2] explored blood as a two-phase fluid made of plasma and suspended RBCs, which significantly affects its flow in both healthy and diseased conditions. Their study demonstrated how this two-phase model can explain variations in RBC distribution, especially in constricted arteries. Many researchers have explored blood flow in

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stenosed arteries, including studies on magnetic effects, overlapping stenosis and the influence of inclination [3-7]. Stenosis is a small, plaque-like structure that forms on arterial walls due to deposits of cholesterol, fats, and other substances. This narrowing of the artery increases resistance and alters blood flow patterns. Recent studies have also focused on advanced factors. Abdelsalam *et al.*, [8] explored the flow of nanofluid in diseased arteries with stenosis and aneurysms, considering the effects of electroosmotic forces and nanoparticle size. Zidan *et al.*, [9] analyzed entropy generation in blood flow through a catheterized stenosed artery. Gandhi *et al.*, [10] investigated how different nanoparticle shapes influence heat radiation and entropy in multi-stenosed arteries. Shahzadi *et al.* [11] studied the impact of electroosmotic forces on stenosed aneurysmal artery using a fractional second-grade fluid model with ternary nanoparticles (Cu, Ag, CuO). Jamali *et al.*, [12] found that stenosis increases resistance, significantly reducing blood flow to vital organs, potentially causing symptoms of atherosclerosis, such as stroke, blood clots, or heart attacks.

The use of magnetic fields has opened new possibilities in controlling blood flow, particularly for therapeutic purposes. Magnetohydrodynamic (MHD) explores how magnetic forces interact with electrically conducting fluids like blood. Early work by [13,14] explored theoretical effects of magnetic fields, while more recent models by Tashtoush *et al.*, [15], Sankar and Viswanathan [16] and Kumawat *et al.*, [17]. analyzed MHD flow in various artery geometries. The introduction of nanoparticles into MHD blood models has added a new dimension to hemodynamic studies. Gold nanoparticles (GNPs) are widely studied due to their biocompatibility, thermal conductivity and role in targeted drug delivery as reported by Yu *et al.*, [18]. Several researchers [19-21] have demonstrated that gold nanoparticles (GNPs) can enhance flow behavior, minimize temperature irregularities, and facilitate efficient drug transport in diseased arteries. Kumawat *et al.* [22] extended previous studies by examining MHD effects in permeable curved arteries with varying viscosity and radiation, identifying arterial wall permeability and curvature as critical factors in atherosclerosis. Abbas *et al.*, [23] analyzed MHD effects on Casson fluid under the Soret effect, which involves temperature-driven mass transfer, offering further insights into the interplay of magnetic fields and blood flow.

The Casson fluid model is widely used for simulating the non-Newtonian properties of blood, particularly its shear-thinning behavior at low shear rates. The Casson fluid model has been employed to address the rheological complexity of blood, as it better represents shear-thinning behavior at low shear and has been widely applied to simulate realistic blood flow in porous or constricted arteries, as shown in Walawender *et al.*, [24], Maiti *et al.*, [25] and Raje *et al.*, [26]. Jamil *et al.*, [27] utilized the Casson fluid model to analyze magnetohydrodynamic (MHD) effects in inclined stenosed artery demonstrating its relevance in modeling real arterial conditions. Alebraheem and Ramzan [28] further investigated swirling Casson nanofluid flow with magnetic field effects. In a more recent study, Maiti *et al.* [25] used the Casson fluid model to analyze blood flow in porous arteries, incorporating fractional derivatives and the effects of heat transfer, thermal radiation, body acceleration and blood concentration. Walelign *et al.*, [29] analyzed Casson nanofluid flow over an inclined cylinder under magnetic field, chemical reactions, and heat transfer. Farooq *et al.*, [30] compared Casson nanofluid flow in stretching cylinders and plates, while Raje *et al.*, [26] examined the effect of Casson parameters on temperature profiles in porous cylinders.

The Caputo-Fabrizio fractional derivative is a type of fractional derivative that does not include a singular kernel. This model has become vital for accurately representing blood flow especially in capturing long-term memory effects and provides more stable solutions in time-dependent system. Studies by Atangana *et al.*, [31], Moitai *et al.*, [32] and Sheikh *et al.*, [33] have shown that fractional-order models can significantly enhance the prediction of blood velocity and temperature profiles in cylindrical and stenosed arteries. Similarly, Jamil *et al.*, [34] employed Caputo-Fabrizio fractional derivatives to study the flow of Casson fluid in a constricted artery and found that fractional

derivatives produced highly accurate results. Imtiaz *et al.*, [35] compared the Caputo-Fabrizio and Atangana-Baleanu fractional derivatives in modeling blood flow through a cylindrical tube, focusing on velocity distribution and the role of magnetic nanoparticles as carriers. Shah *et al.*, [36] explored the impact of fractional-order derivatives and magnetic fields on blood flow by replacing the first-order time derivatives with Caputo fractional derivatives. This approach was further expanded by researchers like Tabi *et al.*, [37] and Maiti *et al.*, [25], who studied blood flow behavior in arteries using Caputo-Fabrizio fractional derivatives. Yadeta *et al.*, [38] emphasized the importance of studying blood flow with fractional derivatives for treating conditions like atherosclerosis and hypertension.

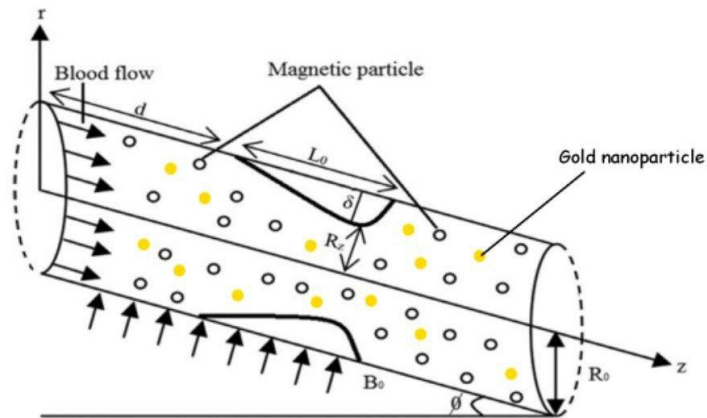
Nanoparticles play a crucial role in enhancing blood flow modeling and medical applications due to their unique properties. Khanduri and Sharma [39] studied the effects of hybrid nanoparticles on blood circulation in constricted arteries, highlighting their significance in fluid dynamics. Similarly, Shahzad *et al.*, [40] investigated hybrid nanoparticles under electroosmotic parameters, demonstrating their potential to address flow irregularities in stenosed arteries. According to the studies by Hamad *et al.*, [41] nanoparticles are useful because they are safe for the body, can break down naturally, have good stability, and conduct heat well, making them suitable for addressing blood flow problems. Studies by Jalili *et al.*, [42] have shown that gold nanoparticles improve blood flow and regulate artery temperature, which are critical for treating conditions like atherosclerosis. Waqas *et al.*, [43] demonstrated the applications of these nanoparticles in therapies such as photothermal therapy and immunotherapy. Advanced models have incorporated gold nanoparticles to simulate blood flow dynamics more accurately. For example, Imoro *et al.*, [21] used Caputo fractional derivatives to study the effects of thermal radiation and chemical reactions on hybrid nanofluids with gold and copper nanoparticles. Gold nanoparticles have also shown significant promise in drug delivery and flow control.

Many studies have examined Casson fluid flow, magnetic fields, fractional calculus, and gold nanoparticles separately, but only a few have integrated these aspects within the context of inclined stenosed arteries. This gap is significant because advancing the understanding of blood flow characteristics requires the development of more precise mathematical models using the fractional derivative approach, which can provide deeper insights into complex hemodynamics. To address this gap, the present study develops a mathematical model for unsteady, non-Newtonian Casson blood flow with suspended gold nanoparticles in an inclined stenosed artery, incorporating magnetic field effects and employing Caputo–Fabrizio fractional derivatives. The main objectives are to analyse the influence of magnetic fields on Casson fluid flow and to evaluate the role of gold nanoparticles in enhancing flow characteristics under these conditions, with simulations carried out using Mathcad.

## 2. Formulation of the problem

### 2.1 Geometry of Stenosis

This study models the unsteady, incompressible flow of non-Newtonian Casson fluid through an inclined stenosed artery under the influence of a magnetic field. The model incorporates gold nanoparticles and applies the Caputo-Fabrizio fractional derivative to account for memory effects in blood flow. Figure 1 shows the geometry of an inclined artery with a symmetrical stenosis. The artery is assumed to be cylindrical, and the narrowing (stenosis) is modeled using a cosine function to reflect a smooth, gradual blockage along the artery wall. The blood is assumed to flow along the z-axis, while the r-axis represents the radial direction. The inclination of the artery is considered to reflect realistic body posture. This geometry is used to observe how blood and magnetic particles behave when flowing through a partially blocked and inclined vessel.



**Fig. 1.** Geometry figure of an inclined stenosed blood flow

The governing equations are the Navier-Stokes equations describing the blood flow, the Maxwell's relations describing the magnetic field and the Newton's second law describing the particle motion.

The Maxwell equations are

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{B} = \mu_0 \vec{J}, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (1)$$

A pressure gradient is required to drive fluid through a channel or heart valve, and it is given in [9] as follows:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega_p t) \quad (2)$$

where  $A_0$ , which is the constant part of the pressure gradient, and  $A_1$  being the amplitude of the fluctuating component of pressure gradient which is responsible for systolic and diastolic pressures. The governing continuity and momentum equations of blood mixed with magnetic particles in the cylindrical coordinates  $(r, \theta, z)$  are provided by Maiti *et al.*, [25] and Raje *et al.*, [26]

$$\begin{aligned} \rho_{nf} \frac{\partial u(r, t)}{\partial t} = & -\frac{\partial p}{\partial z} + \mu_{nf} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, t)}{\partial r}\right) \\ & + KN(v(r, t) - u(r, t)) - \sigma_{nf}^2 B^2 \sin(\theta) u(r, t) + g \sin(\phi), A_0 > 0 \end{aligned} \quad (3)$$

and the motion of magnetic particle

$$m \frac{\partial v(r, t)}{\partial t} = K(u(r, t) - v(r, t)) \quad (4)$$

where  $A_0$  and  $A_1$  are the pulsatile components of the pressure gradient that cause systolic and diastolic pressure.  $\rho_{nf}$  is the fluid density,  $\beta$  is the material parameter of Casson fluid,  $\mu_{nf}$  is the

dynamic viscosity,  $p$  is the pressure,  $K$  is the Stokes constant,  $N$  is the number of magnetic particles.  $B$  is the applied magnetic field.

The initial and boundary conditions in the blood flow inside a circular cylinder with radius  $R_0$  are:

$$\begin{aligned} u(r, t) = 0, v(r, 0) = 0, r \in [0, R_0], \\ u(R_0, t) = 0, v(R_0, t) = 0, t > 0. \end{aligned} \quad (5)$$

To construct the non-dimensional form of the current model, the following variables are introduced:

$$\begin{aligned} r^* = \frac{r}{R_0}, \quad t^* = \frac{tv}{R_0^2}, \quad u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{v_0}, \\ z^* = \frac{z}{R_0}, \quad p^* = \frac{pR_0}{\mu u_0}, \quad g^* = \frac{g}{u_0^2/R_0} \end{aligned} \quad (6)$$

By replacing  $\frac{\partial}{\partial t} = D_t^\alpha$  into Eq. (3) and Eq. (4), non-dimensional parameter in Eq. (6) and applying the boundary conditions from Eq. (5), the non-dimensional governing equation is obtained as follows:

$$\begin{aligned} D_t^\alpha u(r, t) = b_5(A_0 + A_1 \cos(\omega t)) + b_6\beta_1 \left( \frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, t)}{\partial r} \right) \\ + b_5 R(v(r, t) - u(r, t)) - b_8 M u(r, t) + \frac{1}{b_5} \left( \frac{\sin \phi}{F} \right), A_0 > 0 \end{aligned} \quad (7)$$

$$G \frac{\partial v(r, t)}{\partial t} = u(r, t) - v(r, t), \quad (8)$$

$$\begin{aligned} \left( \frac{r}{R_z}, 0 \right) = 0, v \left( \frac{r}{R_z}, 0 \right) = 0, \text{ at } \frac{r}{R_z} \in [0, 1], \\ u(1, t) = 0, v(1, t) = 0, \text{ at } \frac{r}{R_z} = 1, t > 0. \end{aligned} \quad (9)$$

where  $\beta_1 = \left(1 + \frac{1}{\beta}\right)$  is constant parameter,  $R = \frac{r_0^2 KN}{\mu_f}$  is particle concentration parameter, Hartmann number,  $M = \frac{\sigma_f r_0^2 B_0^2 \sin(\theta)}{\mu_f}$ .  $b_5 = \frac{1}{(1-\phi)+\phi \frac{\rho_s}{\rho_f}}$ ,  $b_6 = \frac{b_5}{(1-\phi)^{2.5}}$ ,  $b_4 = 1 + \frac{3(\sigma-1)\phi}{(\sigma+2)+(\sigma-1)\phi}$ ,  $G = \frac{mv}{K r_0^2}$  is the mass parameter of magnetic particles, and lastly,  $F = \frac{\mu_f}{b_5 u_0 r_0 g}$  is the inclination angle parameter. To simplify, after applying Laplace transform to the Eqs. (7) and (8) along with the boundary conditions Eq. (9), the fluid flow model can be expressed as below:

$$\frac{s\bar{u}(r, s)}{s + \alpha(1 + s)} = \frac{A_0}{s} + \frac{A_1 s}{s^2 + \omega^2} + \beta_1 \left( \frac{\partial^2 \bar{u}(r, s)}{\partial s} + \frac{1}{r} \frac{\partial \bar{u}(r, s)}{\partial r} \right)$$

$$+R\{\bar{v}(r, s) - \bar{u}(r, s)\} - M\bar{u}(r, s) + \frac{\sin(\phi)}{sF} \quad (10)$$

$$G \frac{s\bar{v}(r, s)}{s + \alpha(1 - s)} = \bar{u}(r, s) - \bar{v}(r, s) \quad (11)$$

$$\bar{u}(1, s) = 0, \bar{v}(1, s) = 0 \quad (12)$$

From Eq. (11), the following equation can be obtained

$$\bar{v}(r, s) = \frac{s + \alpha(1 - s)}{s + sG + \alpha(1 - s)} \bar{u}(r, s) \quad (13)$$

Substituting  $\bar{v}(r, s)$  from Eq. (13) into Eq. (10):

$$\begin{aligned} \bar{u}(r, s) & \left( \frac{s}{s + \alpha(1 - s)} - R \left( \frac{s + \alpha(1 - s)}{s + sG + \alpha(1 - s)} \right) + M + R \right) \\ & = \frac{A_0}{s} + \frac{A_1 s}{s^2 + \omega^2} + \beta_1 \left( \frac{\partial^2 \bar{u}(r, s)}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}(r, s)}{\partial r} \right) + \frac{\sin(\phi)}{sF} \end{aligned} \quad (14)$$

Applying the finite Hankel and considering the given boundary conditions in Eq. (12), we get:

$$\begin{aligned} \bar{u}_H(r_n, s) & \left( \frac{s}{s + \alpha(1 - s)} - R \left( \frac{s + \alpha(1 - s)}{s + sG + \alpha(1 - s)} \right) + M + R \right) \\ & = \left( \frac{A_0}{s} + \frac{A_1 s}{s^2 + \omega^2} + \frac{\sin(\phi)}{sF} \right) \frac{J_1(r_n)}{r_n} - \beta_1 r_n^2 \bar{u}_H(r_n, s) \end{aligned} \quad (15)$$

The term  $\bar{u}_H(r_n, s)$  is the finite Hankel transform of  $\bar{u}(r, s)$ , defined by the integral  $\bar{u}_H(r_n, s) = \int_0^1 r \bar{u}(r, s) J_0(r_n r) dr$  where  $\bar{u}(r, s) = \mathcal{L}[u(r, t)]$  and  $r_n n = 1, 2, \dots$  are the positive roots of the equation  $J_0(x) = 0$ .  $J_0$  is the the Bessel function of order zero of the first kind. Using Eq. (15), we simplify the coefficient of  $\bar{u}_H(r_n, s)$  so that we obtain

$$\bar{u}_H(r_n, s) = \frac{s^2 y_{5n} + s y_{6n} + \alpha^2}{s^2 y_{2n} + s y_{3n} + s y_{4n}} \left[ \frac{1}{s} \left( A_0 + \frac{\sin(\phi)}{sF} \right) + \frac{A_1 s}{s^2 + \omega^2} \right] \frac{J_1(r_n)}{r_n} \quad (16)$$

$$\bar{u}_H(r_n, s) = \left( \frac{y_{9n}}{s - y_{7n}} + \frac{y_{10n}}{s - y_{8n}} \right) \left[ \frac{1}{s} \left( A_0 + \frac{\sin(\phi)}{sF} \right) + \frac{A_1 s}{s^2 + \omega^2} \right] \frac{J_1(r_n)}{r_n} \quad (17)$$

The parameters used to simplifying the coefficient of  $\bar{u}_H(r_n, s)$  as presented in Eq. (16) and Eq. (17) are

$$\begin{aligned} y_{1n} &= H a^2 + R + \frac{r_n}{Re} \\ y_{2n} &= 1 + G - \alpha - R - R a^2 + 2 R \alpha + y_{1n} + \alpha^2 y_{1n} - 2 \alpha y_{1n} - G y_{1n} - G \alpha y_{1n} \end{aligned}$$

$$\begin{aligned}
 y_{3n} &= \alpha + 2R\alpha^2 - 2R\alpha - 2y_{1n}\alpha^2 + 2\alpha y_{1n} + G\alpha y_{1n} \\
 y_{4n} &= \alpha^2 y_{1n} - R\alpha^2 \\
 y_{5n} &= 1 + \alpha^2 - 2\alpha + G - G\alpha \\
 y_{6n} &= -2\alpha^2 + 2\alpha + G\alpha \\
 y_{7n} &= \frac{-y_{3n} + \sqrt{y_{3n}^2 - 4y_{2n}y_{4n}}}{2y_{2n}} \\
 y_{8n} &= \frac{-y_{3n} - \sqrt{y_{3n}^2 - 4y_{2n}y_{4n}}}{2y_{2n}} \\
 y_{9n} &= \frac{y_{7n}^2 y_{5n} + y_{7n} y_{6n} + \alpha^2}{y_{7n} - y_{8n}} \\
 y_{10n} &= \frac{y_{8n}^2 y_{5n} + y_{8n} y_{6n} + \alpha^2}{y_{8n} - y_{7n}}
 \end{aligned} \tag{18}$$

The Laplace transform of the image function  $\bar{u}_H(r_n, s)$  described in Eq. (17) is derived using the Robotnov and Hartley functions.

$$\begin{aligned}
 \bar{u}_H(r_n, s) &= \frac{J_1(r_n)}{r_n} \left[ (e^{y_{7n}t} - 1) \left( \frac{A_0 y_{9n}}{y_{7n}} + \frac{y_{9n} \sin \phi}{y_{7n} F} \right) \right. \\
 &\quad + (e^{y_{8n}t} - 1) \left( \frac{A_0 y_{10n}}{y_{8n}} + \frac{y_{10n} \sin \phi}{y_{8n} F} \right) + y_{9n} A_1 e^{y_{7n}t} * \cos \omega t + y_{10n} A_1 e^{y_{8n}t} \\
 &\quad \left. * \cos \omega t \right]
 \end{aligned} \tag{19}$$

The Eq. (19) can be obtained by using the inverse Hankel transform.

$$u(r, t) = 2 \sum_{n=1}^{\infty} \frac{J_0\left(\frac{r}{R_z} \zeta_n\right)}{J_1^2(\zeta_n)} \times \bar{u}_H(\zeta_n, s) \tag{20}$$

The velocity of the magnetic particles mixed with blood is obtained from Eq. (13)

$$\begin{aligned}
 u(r, t) &= 2 \sum_{n=1}^{\infty} \frac{J_0\left(\frac{r}{R_z} r_n\right)}{r_n J_1(r_n)} \left[ (e^{y_{7n}t} - 1) \left( \frac{A_0 y_{9n}}{y_{7n}} + \frac{y_{9n} \sin \phi}{y_{7n} F} \right) \right. \\
 &\quad + (e^{y_{8n}t} - 1) \left( \frac{A_0 y_{10n}}{y_{8n}} + \frac{y_{10n} \sin \phi}{y_{8n} F} \right) + y_{9n} A_1 e^{y_{7n}t} \cos \omega t \\
 &\quad \left. + y_{10n} A_1 e^{y_{8n}t} * \cos \omega t \right]
 \end{aligned} \tag{21}$$

The velocity of the magnetic particles mixed with blood is obtained from Eq. (13)

$$v(r, t) = y_{12n}(1 - y_{11n})[u(r, t) * e^{-y_{12n}t}], \text{ for } 0 < \alpha \leq 1 \quad (22)$$

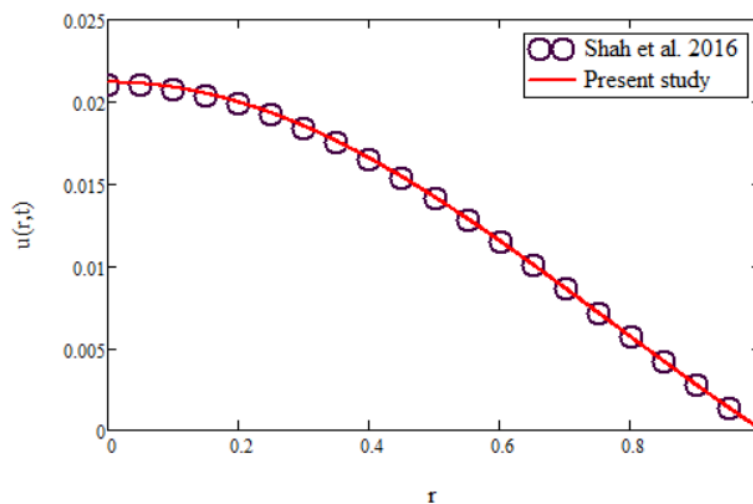
The parameters presented in Eq. (22) are as follows:

$$m_{11n} = \frac{1 - \alpha}{G - \alpha + 1} \quad (23)$$

$$m_{12n} = \frac{\alpha}{G - \alpha + 1} \quad (24)$$

### 3. Results and discussion

This section presents the numerical results and discussion of the analytical study on non-Newtonian magnetic Casson blood flow with gold nanoparticles through an inclined stenosed artery. The governing equations for the velocity profiles of both blood flow  $u(r, t)$  and magnetic nanoparticle  $v(r, t)$  are solved using Mathcad software based on the Caputo-Fabrizio fractional derivative approach. The effects of key parameters such as Hartmann number  $Ha$ , fractional order  $\alpha$ , Casson parameter  $\beta$ , and time  $t$  are analyzed in detail. For each graph, all other parameters were kept constant with values  $A_0 = 0.1, A_1 = 0.1, G = 0.7, R = 0.5, \omega = \frac{\pi}{4}, t = 0.25, \beta = 0.4$  and  $Ha = 2$ . To validate the current model, Figure 2 presents the result for  $\alpha = 1$ , where the solution reduces to the classical model. The result was compared with those obtained by [26] confirming the accuracy of the analytical approach involving Laplace and finite Hankel transforms with Caputo-Fabrizio fractional derivative.



**Fig. 2.** Comparison of velocity distributions between Shah *et al.*, [36] and present study

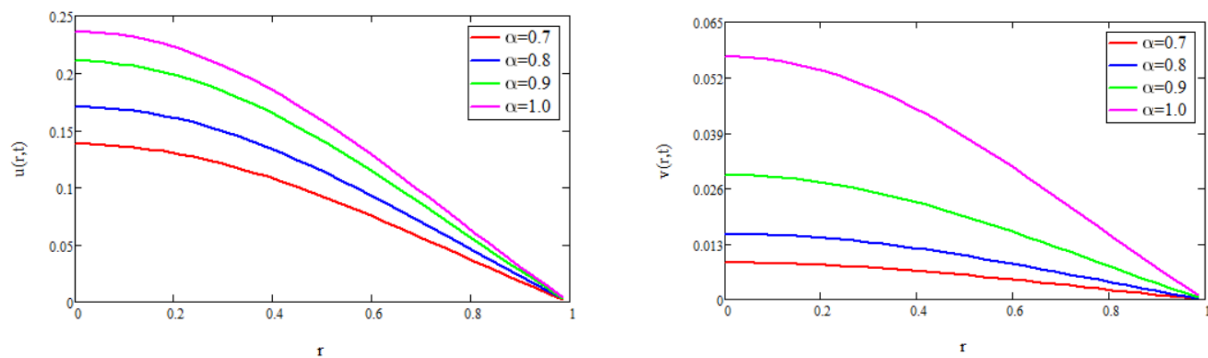
**Table 1**

Thermophysical parameters of blood and gold nanoparticles

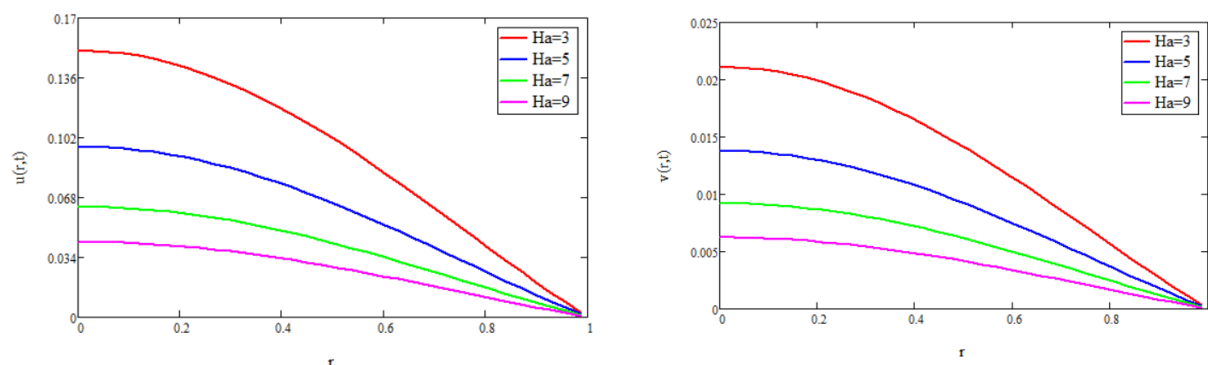
Thermophysical properties	Blood	Gold
$\rho$ (kg/m <sup>3</sup> )	1063	19300
$c_p$ (J/kg K)	3594	129
$\rho$ (kg/m <sup>3</sup> )	0.492	318
$\alpha$ (S/m)	0.8	$4.1 \times 10^7$

Figure 3 show the influence of the fractional parameter  $\alpha = 0.7, 0.8, 0.9, 1.0$  on the axial velocities. As  $\alpha$  increases, both blood flow velocity  $u(r, t)$  and magnetic particle velocity  $v(r, t)$  also increase. This result demonstrates that higher memory effect (lower  $\alpha$ ) slows down the motion of fluid and particles due to increased resistance. The case  $\alpha = 1$  represents the classical derivative, producing the highest velocity. For Figure 4, the axial velocities decrease as the Hartmann number increases. A higher  $Ha$  value means a stronger magnetic field and thus more resistance to flow. The magnetic field is found to lower the axial velocities of both blood and magnetic particles. This happens because the stronger Lorentz force works against the flow, slowing it down within the system.

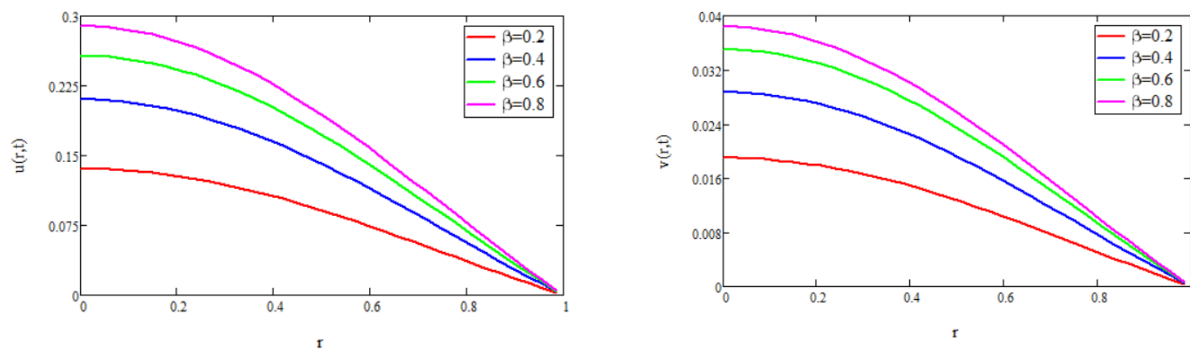
The effect of the Casson parameter  $\beta$ , which represents the yield stress of the Casson fluid, is depicted in Figure 5. Both blood velocity and magnetic particles increase when the Casson fluid parameters increase. This result is consistent with the findings of [25] who studied flow over a horizontal cylinder. This hypothesis suggests that an increase in  $\beta$  leads to a thinner boundary layer. Figure 6 shows the effect of time level on the blood and magnetic distributions. It shows that both profiles increase as time increase. The study reveals that the fractional parameters has a significant impact on controlling the distribution of blood flow. This result supports the observations made by [12], who emphasized the importance of non-singular kernel operators in modeling biological systems.



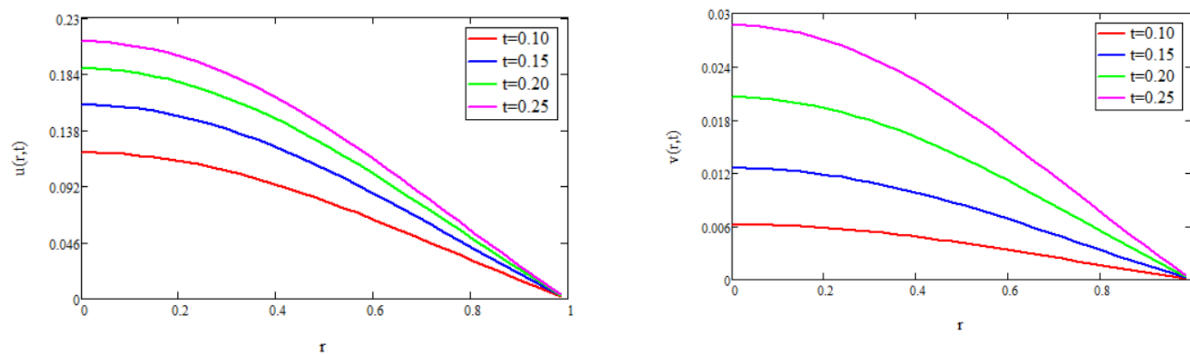
**Fig. 3.** Variation of axial velocities  $u(r,t)$  and  $v(r,t)$  for different values of fractional parameters,  $\alpha$  for  $A_0 = 0.075$ ,  $A_1 = 0.075$ ,  $G = 0.7$ ,  $\omega = \frac{\pi}{4}$ ,  $Ha = 1$ ,  $t = 0.25$ ,  $\beta = 0.4$  against  $r$



**Fig. 4.** Variation of axial velocities  $u(r,t)$  and  $v(r,t)$  for different values of Hartmann numbers,  $Ha$  for  $A_0 = 0.075$ ,  $A_1 = 0.075$ ,  $G = 0.7$ ,  $\omega = \frac{\pi}{4}$ ,  $Ha = 1$ ,  $t = 0.25$ ,  $\beta = 0.4$  against  $r$



**Fig. 5.** Variation of axial velocities  $u(r,t)$  and  $v(r,t)$  for different values of Casson parameters  $\beta$  for  $A_0 = 0.075$ ,  $A_1 = 0.075$ ,  $G = 0.7$ ,  $\omega = \frac{\pi}{4}$ ,  $Ha = 1$ ,  $t = 0.25$ ,  $Ha = 1.0$  against  $r$



**Fig. 6.** Variation of axial velocities  $u(r,t)$  and  $v(r,t)$  for different values of time,  $t$  for  $A_0 = 0.075$ ,  $A_1 = 0.075$ ,  $G = 0.7$ ,  $\omega = \frac{\pi}{4}$ ,  $Ha = 1$ ,  $t = 0.25$ ,  $Ha = 1.0$  against  $r$

#### 4. Conclusions

This study presents a mathematical model to investigate unsteady non-Newtonian Casson blood flow with gold nanoparticles through an inclined stenosed artery under the influence of magnetic fields. The model incorporates Caputo-Fabrizio fractional derivatives, Laplace and Hankel transforms, and validates the velocity profiles against previous studies. The results show that increasing the fractional order enhances both blood and particle velocities due to reduced memory effects. Conversely, higher Hartmann numbers reduce velocity as magnetic resistance increases. An increase in the Casson parameter leads to higher velocity, indicating a shift towards Newtonian fluid behaviour. Additionally, velocity profiles become more developed over time. These findings suggest that fractional calculus and gold nanoparticles play a significant role in improving blood flow characteristics, which may aid future biomedical and drug delivery applications in stenosed arteries.

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